

The Ontario Curriculum  
Grade 11

*REVISED*

# Mathematics

2006



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This publication is available on the Ministry of Education website at  
<http://www.edu.gov.on.ca>.

*Every effort has been made in this publication to identify mathematics resources and tools in generic terms. In cases where a particular product is used by teachers in schools across Ontario, that product is identified by its trade name, in the interests of clarity. Reference to particular products in no way implies an endorsement of those products by the Ministry of Education.*

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# Introduction

This document replaces the Grade 11 courses found in the *The Ontario Curriculum, Grades 11 and 12: Mathematics, 2000*. Beginning in September 2006, all Grade 11 mathematics courses will be based on the expectations outlined in this document.

## Secondary Schools for the Twenty-First Century

The goal of Ontario secondary schools is to support high-quality learning while giving individual students the opportunity to choose programs that suit their skills and interests. The updated Ontario curriculum, in combination with a broader range of learning options outside traditional classroom instruction, will enable students to better customize their high school education and improve their prospects for success in school and in life.

## The Place of Mathematics in the Curriculum

The unprecedented changes that are taking place in today's world will profoundly affect the future of today's students. To meet the demands of the world in which they will live, students will need to adapt to changing conditions and to learn independently. They will require the ability to use technology effectively and the skills for processing large amounts of quantitative information. Today's mathematics curriculum must prepare students for their future roles in society. It must equip them with essential mathematical knowledge and skills; with skills of reasoning, problem solving, and communication; and, most importantly, with the ability and the incentive to continue learning on their own. This curriculum provides a framework for accomplishing these goals.

The choice of specific concepts and skills to be taught must take into consideration new applications and new ways of doing mathematics. The development of sophisticated yet easy-to-use calculators and computers is changing the role of procedure and technique in mathematics. Operations that were an essential part of a procedures-focused curriculum for decades can now be accomplished quickly and effectively using technology, so that students can now solve problems that were previously too time-consuming to attempt, and can focus on underlying concepts. "In an effective mathematics program, students learn in the presence of technology. Technology should influence the mathematics content taught and how it is taught. Powerful assistive and enabling computer and handheld technologies should be used seamlessly in teaching, learning, and assessment."<sup>1</sup> This curriculum integrates appropriate technologies into the learning and doing of mathematics, while recognizing the continuing importance of students' mastering essential numeric and algebraic skills.

Mathematical knowledge becomes meaningful and powerful in application. This curriculum embeds the learning of mathematics in the solving of problems based on real-life situations. Other disciplines are a ready source of effective contexts for the study of mathematics. Rich problem-solving situations can be drawn from related disciplines, such as computer science,

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1. Expert Panel on Student Success in Ontario, *Leading Math Success: Mathematical Literacy, Grades 7–12 – The Report of the Expert Panel on Student Success in Ontario, 2004* (Toronto: Ontario Ministry of Education, 2004), p. 47. (Referred to hereafter as *Leading Math Success*.)

business, recreation, tourism, biology, physics, or technology, as well as from subjects historically thought of as distant from mathematics, such as geography or art. It is important that these links between disciplines be carefully explored, analysed, and discussed to emphasize for students the pervasiveness of mathematical knowledge and mathematical thinking in all subject areas.

The development of mathematical knowledge is a gradual process. A coherent and continuous program is necessary to help students see the “big pictures”, or underlying principles, of mathematics. The fundamentals of important skills, concepts, processes, and attitudes are initiated in the primary grades and fostered through elementary school. The links between Grade 8 and Grade 9 and the transition from elementary school mathematics to secondary school mathematics are very important in the student’s development of confidence and competence.

The secondary courses are based on principles that are consistent with those that underpin the elementary program, facilitating the transition from elementary school. These courses reflect the belief that students learn mathematics effectively when they are initially given opportunities to investigate ideas and concepts and are then guided carefully into an understanding of the abstract mathematics involved. Skill acquisition is an important part of the program; skills are embedded in the contexts offered by various topics in the mathematics program and should be introduced as they are needed.

### **Roles and Responsibilities in Mathematics Programs**

**Students.** Students have many responsibilities with regard to their learning. Students who make the effort required to succeed in school and who are able to apply themselves will soon discover that there is a direct relationship between this effort and their achievement, and will therefore be more motivated to work. There will be some students, however, who will find it more difficult to take responsibility for their learning because of special challenges they face. For these students, the attention, patience, and encouragement of teachers and family can be extremely important factors for success. However, taking responsibility for their own progress and learning is an important part of education for all students, regardless of their circumstances.

Mastery of concepts and skills in mathematics requires a sincere commitment to work and study. Students are expected to develop strategies and processes that facilitate learning and understanding in mathematics. Students should also be encouraged to actively pursue opportunities to apply their problem-solving skills outside the classroom and to extend and enrich their understanding of mathematics.

**Parents.** Parents have an important role to play in supporting student learning. Studies show that students perform better in school if their parents or guardians are involved in their education. By becoming familiar with the curriculum, parents can find out what is being taught in the courses their children are taking and what their children are expected to learn. This awareness will enhance parents’ ability to discuss their children’s work with them, to communicate with teachers, and to ask relevant questions about their children’s progress. Knowledge of the expectations in the various courses also helps parents to interpret teachers’ comments on student progress and to work with them to improve student learning.

The mathematics curriculum promotes lifelong learning. In addition to supporting regular school activities, parents can encourage their sons and daughters to apply their problem-solving skills to other disciplines and to real-world situations. Attending parent-teacher interviews, participating in parent workshops, becoming involved in school council activities (including becoming a school council member), and encouraging students to complete their assignments at home are just a few examples of effective ways to support student learning.

**Teachers.** Teachers and students have complementary responsibilities. Teachers are responsible for developing appropriate instructional strategies to help students achieve the curriculum expectations for their courses, as well as for developing appropriate methods for assessing and evaluating student learning. Teachers also support students in developing the reading, writing, and oral communication skills needed for success in their mathematics courses. Teachers bring enthusiasm and varied teaching and assessment approaches to the classroom, addressing different student needs and ensuring sound learning opportunities for every student.

Recognizing that students need a solid conceptual foundation in mathematics in order to further develop and apply their knowledge effectively, teachers endeavour to create a classroom environment that engages students' interest and helps them arrive at the understanding of mathematics that is critical to further learning.

Using a variety of instructional, assessment, and evaluation strategies, teachers provide numerous opportunities for students to develop skills of inquiry, problem solving, and communication as they investigate and learn fundamental concepts. The activities offered should enable students not only to make connections among these concepts throughout the course but also to relate and apply them to relevant societal, environmental, and economic contexts. Opportunities to relate knowledge and skills to these wider contexts – to the goals and concerns of the world in which they live – will motivate students to learn and to become lifelong learners.

**Principals.** The principal works in partnership with teachers and parents to ensure that each student has access to the best possible educational experience. To support student learning, principals ensure that the Ontario curriculum is being properly implemented in all classrooms using a variety of instructional approaches. They also ensure that appropriate resources are made available for teachers and students. To enhance teaching and learning in all subjects, including mathematics, principals promote learning teams and work with teachers to facilitate participation in professional development. Principals are also responsible for ensuring that every student who has an Individual Education Plan (IEP) is receiving the modifications and/or accommodations described in his or her plan – in other words, for ensuring that the IEP is properly developed, implemented, and monitored.



# The Program in Mathematics

## Overview of the Program

The Grade 11 mathematics program builds on the Grade 9 and 10 program, relying on the same fundamental principles on which that program was based. Both are founded on the premise that students learn mathematics most effectively when they have a thorough understanding of mathematical concepts and procedures, and when they build that understanding through an investigative approach. This curriculum is designed to help students build a solid conceptual foundation in mathematics that will enable them to apply their knowledge and skills and further their learning successfully.

An important part of every course in the mathematics program is the process of inquiry, in which students develop methods for exploring new problems or unfamiliar situations. Knowing how to learn mathematics is the underlying expectation that every student in every course needs to achieve. An important part of the inquiry process is that of taking the conditions of a real-world situation and representing them in mathematical form. A mathematical representation can take many different forms – for example, it can be a physical model, a diagram, a graph, a table of values, an equation, or a computer simulation. It is important that students understand the relationships between the various representations of a given situation and that they become familiar with increasingly sophisticated mathematical representations as they progress through secondary school.

The choice of some specific concepts and skills in the curriculum has taken into consideration the development of sophisticated yet easily used calculators and computer software. The curriculum has been designed to integrate appropriate technologies into the learning and doing of mathematics, while equipping students with the manipulation skills necessary to understand other aspects of the mathematics that they are learning, to solve meaningful problems, and to continue to learn mathematics with success in the future. Technology is not used to replace skill acquisition. Rather, technology is required when its use represents either the only way or the most effective way to achieve an expectation.

Like the earlier curriculum experienced by students, the senior secondary curriculum adopts a strong focus on the processes that best enable students to understand mathematical concepts and learn related skills. Attention to the mathematical processes is considered to be essential to a balanced mathematics program. The seven mathematical processes identified in this curriculum are *problem solving*, *reasoning and proving*, *reflecting*, *selecting tools and computational strategies*, *connecting*, *representing*, and *communicating*. Each of the Grade 11 mathematics courses includes a set of expectations – referred to in this document as the “mathematical process expectations” – that outline the knowledge and skills involved in these essential processes. The mathematical processes apply to student learning in all areas of a mathematics course.

A balanced mathematics program at the secondary level includes the development of algebraic skills. This curriculum has been designed to equip students with the algebraic skills they need to understand other aspects of mathematics that they are learning, to solve meaningful problems, and to continue to meet with success as they study mathematics in the future. The algebraic skills required in each course have been carefully chosen to support the other topics included in



the course. Calculators and other appropriate technology will be used when the primary purpose of a given activity is the development of concepts or the solving of problems, or when situations arise in which computation or symbolic manipulation is of secondary importance.

**Courses in Grade 11.** Four types of courses are offered in the mathematics program in Grade 11: *university preparation*, *university/college preparation*, *college preparation*, and *workplace preparation*. Students choose course types on the basis of their interests, achievement, and postsecondary goals. The course types are defined as follows:

- **University preparation** courses are designed to equip students with the knowledge and skills they need to meet the entrance requirements for university programs.
- **University/college preparation** courses are designed to equip students with the knowledge and skills they need to meet the entrance requirements for specific programs offered at universities and colleges.
- **College preparation** courses are designed to equip students with the knowledge and skills they need to meet the requirements for entrance to most college programs or for admission to apprenticeship or other training programs.
- **Workplace preparation** courses are designed to equip students with the knowledge and skills they need to meet the expectations of employers, if they plan to enter the workplace directly after graduation, or the requirements for admission to certain apprenticeship or other training programs.

#### Courses in Mathematics, Grade 11

Course Name	Course Type	Course Code	Prerequisite
Functions	University	MCR3U	Grade 10 Principles of Mathematics, Academic
Functions and Applications	University/College	MCF3M	Grade 10 Principles of Mathematics, Academic, or Grade 10 Foundations of Mathematics, Applied
Foundations for College Mathematics	College	MBF3C	Grade 10 Foundations of Mathematics, Applied
Mathematics for Work and Everyday Life	Workplace	MEL3E	Grade 9 Mathematics, Academic or Applied, or a ministry-approved locally developed Grade 10 mathematics course

*Note:* Each of the courses listed above is worth one credit.

**Half-Credit Courses.** The courses outlined in this document are designed to be offered as full-credit courses. However, they may also be delivered as half-credit courses.

Half-credit courses, which require a minimum of fifty-five hours of scheduled instructional time, must adhere to the following conditions:

- The two half-credit courses created from a full course must together contain all of the expectations of the full course. The expectations for each half-credit course must be divided in a manner that best enables students to achieve the required knowledge and skills in the allotted time.

- A course that is a prerequisite for another course in the secondary curriculum may be offered as two half-credit courses, but students must successfully complete both parts of the course to fulfil the prerequisite. (Students are not required to complete both parts unless the course is a prerequisite for another course they wish to take.)
- The title of each half-credit course must include the designation *Part 1* or *Part 2*. A half credit (0.5) will be recorded in the credit-value column of both the report card and the Ontario Student Transcript.

Boards will ensure that all half-credit courses comply with the conditions described above, and will report all half-credit courses to the ministry annually in the School October Report.

## Curriculum Expectations

The expectations identified for each course describe the knowledge and skills that students are expected to acquire, demonstrate, and apply in their class work, on tests, and in various other activities on which their achievement is assessed and evaluated.

Two sets of expectations are listed for each *strand*, or broad curriculum area, of each course.

- The *overall expectations* describe in general terms the knowledge and skills that students are expected to demonstrate by the end of each course.
- The *specific expectations* describe the expected knowledge and skills in greater detail. The specific expectations are arranged under subheadings that reflect particular aspects of the required knowledge and skills and that may serve as a guide for teachers as they plan learning activities for their students. The organization of expectations in subgroupings is not meant to imply that the expectations in any subgroup are achieved independently of the expectations in the other subgroups. The subheadings are used merely to help teachers focus on particular aspects of knowledge and skills as they develop and present various lessons and learning activities for their students.

In addition to the expectations outlined within each strand, a list of seven “mathematical process expectations” precedes the strands in all mathematics courses. These specific expectations describe the knowledge and skills that constitute processes essential to the effective study of mathematics. These processes apply to all areas of course content, and students’ proficiency in applying them must be developed in all strands of a mathematics course. Teachers should ensure that students develop their ability to apply these processes in appropriate ways as they work towards meeting the expectations outlined in the strands.

When developing detailed courses of study from this document, teachers are expected to weave together related expectations from different strands, as well as the relevant process expectations, in order to create an overall program that integrates and balances concept development, skill acquisition, the use of processes, and applications.

Many of the expectations are accompanied by examples and/or sample problems, given in parentheses. These examples and sample problems are meant to illustrate the kind of skill, the specific area of learning, the depth of learning, and/or the level of complexity that the expectation entails. Some examples and sample problems may also be used to emphasize the importance of diversity or multiple perspectives. The examples and sample problems are intended only as suggestions for teachers. Teachers may incorporate the examples or sample problems into their lessons, or they may choose other topics, approaches, or problems that are relevant to the expectation.

## Strands

The focus of the strands in each of the Grade 11 courses is briefly described below.

**University Preparation Course.** The Grade 11 university preparation course, Functions, builds on the concepts and skills developed in the Grade 9 and 10 academic mathematics courses. It is designed for students who plan to pursue any one of many university programs, including physics, engineering, social sciences, liberal arts, and education. The concept of functions is explored in all four strands. The Characteristics of Functions strand introduces functions by building on the knowledge and skills developed through the study of linear and quadratic relations in Grades 9 and 10. In working with numerical and algebraic expressions, students develop important algebraic skills and a deeper understanding of equality. Two new functions are introduced in the Exponential Functions and Trigonometric Functions strands. The emphasis is on recognizing real-world applications of these functions, developing the knowledge and skills needed to use the functions, and solving problems. The Discrete Functions strand allows students, through the study of different representations of sequences and series, to revisit patterning and algebra concepts introduced in elementary school. Students learn to connect sequences and series to financial applications involving compound interest and ordinary annuities.

**University/College Preparation Course.** The Grade 11 university/college preparation course, Functions and Applications, shares a focus on functions with the university preparation course, Functions. The Functions and Applications course is designed for students who plan to pursue technology-related programs in college. This course also prepares students for some university programs. Unlike the university preparation course, this course revisits some of the important concepts developed in Grade 10. The first strand, Quadratic Functions, extends knowledge and skills related to quadratics for students who completed the Grade 10 applied mathematics course and reviews this topic for students entering from the Grade 10 academic course. The strand also introduces some of the properties of functions. The other two strands, Exponential Functions and Trigonometric Functions, emphasize real-world applications and help students develop the knowledge and skills needed to solve related problems.

**College Preparation Course.** The Grade 11 college preparation course, Foundations for College Mathematics, includes a blend of topics needed by students who plan to pursue one of a broad range of college programs, including programs in business, human services, hospitality and tourism, and some of the health sciences. To meet the needs of all these students, the course is made up of four strands that address different areas of mathematics. The Mathematical Models strand builds on the concepts connected to linear and quadratic relations developed through the Grade 9 and 10 applied mathematics courses. In this strand, quadratic relations are revisited, and exponential relations are introduced. The Personal Finance strand focuses on compound interest and applications related to investing and borrowing money and owning and operating a vehicle. Applications requiring spatial reasoning are addressed in the Geometry and Trigonometry strand. The fourth strand, Data Management, addresses the practical applications of probability and one-variable statistics.

**Workplace Preparation Course.** The Grade 11 workplace preparation course, Mathematics for Work and Everyday Life, presents a blend of topics dealing with personal finance and with areas of mathematics that are useful in a variety of jobs. The three strands in this course provide

students with the opportunity to solve problems in areas related to Earning and Purchasing; Saving, Investing, and Borrowing; and Transportation and Travel. The gathering and interpretation of data and information is an important aspect of the course. The expectations support the use of hands-on projects and other experiences that make the learning of mathematics more meaningful.

# The Mathematical Processes

Presented at the start of every course in this curriculum document is a set of seven expectations that describe the mathematical processes students need to learn and apply as they work to achieve the expectations outlined within the strands of the course. In the 2000 mathematics curriculum, expectations relating to the mathematical processes were embedded within individual strands. The need to highlight these process expectations arose from the recognition that students should be actively engaged in applying these processes throughout the course, rather than in connection with particular strands.

The mathematical processes that support effective learning in mathematics are as follows:

- problem solving
- reasoning and proving
- reflecting
- selecting tools and computational strategies
- connecting
- representing
- communicating

The mathematical processes are interconnected. Problem solving and communicating have strong links to all the other processes. A problem-solving approach encourages students to reason their way to a solution or a new understanding. As students engage in reasoning, teachers further encourage them to make conjectures and justify solutions, orally and in writing. The communication and reflection that occur during and after the process of problem solving help students not only to articulate and refine their thinking but also to see the problem they are solving from different perspectives. This opens the door to recognizing the range of strategies that can be used to arrive at a solution. By seeing how others solve a problem, students can begin to think about their own thinking (metacognition) and the thinking of others, and to consciously adjust their own strategies in order to make their solutions as efficient and accurate as possible.

The mathematical processes cannot be separated from the knowledge and skills that students acquire throughout the course. Students must problem solve, communicate, reason, reflect, and so on, as they develop the knowledge, the understanding of concepts, and the skills required in the course.

## **Problem Solving**

Problem solving is central to learning mathematics. It forms the basis of effective mathematics programs and should be the mainstay of mathematical instruction. It is considered an essential process through which students are able to achieve the expectations in mathematics, and is an integral part of the mathematics curriculum in Ontario, for the following reasons. Problem solving:

- is the primary focus and goal of mathematics in the real world;
- helps students become more confident mathematicians;

- allows students to use the knowledge they bring to school and helps them connect mathematics with situations outside the classroom;
- helps students develop mathematical understanding and gives meaning to skills and concepts in all strands;
- allows students to reason, communicate ideas, make connections, and apply knowledge and skills;
- offers excellent opportunities for assessing students' understanding of concepts, ability to solve problems, ability to apply concepts and procedures, and ability to communicate ideas;
- promotes the collaborative sharing of ideas and strategies, and promotes talking about mathematics;
- helps students find enjoyment in mathematics;
- increases opportunities for the use of critical-thinking skills (e.g., estimating, classifying, assuming, recognizing relationships, hypothesizing, offering opinions with reasons, evaluating results, and making judgements).

Not all mathematics instruction, however, can take place in a problem-solving context. Certain aspects of mathematics must be explicitly taught. Conventions, including the use of mathematical symbols and terms, are one such aspect, and they should be introduced to students as needed, to enable them to use the symbolic language of mathematics.

**Selecting Problem-Solving Strategies.** Problem-solving strategies are methods that can be used successfully to solve problems of various types. Teachers who use relevant and meaningful problem-solving experiences as the focus of their mathematics class help students to develop and extend a repertoire of strategies and methods that they can apply when solving various kinds of problems – instructional problems, routine problems, and non-routine problems. Students develop this repertoire over time, as they become more mature in their problem-solving skills. By secondary school, students will have learned many problem-solving strategies that they can flexibly use and integrate when faced with new problem-solving situations, or to learn or reinforce mathematical concepts. Common problem-solving strategies include the following: making a model, picture, or diagram; looking for a pattern; guessing and checking; making assumptions; making an organized list; making a table or chart; making a simpler problem; working backwards; using logical reasoning.

### **Reasoning and Proving**

An emphasis on reasoning helps students make sense of mathematics. Classroom instruction in mathematics should always foster critical thinking – that is, an organized, analytical, well reasoned approach to learning mathematical concepts and processes and to solving problems.

As students investigate and make conjectures about mathematical concepts and relationships, they learn to employ *inductive reasoning*, making generalizations based on specific findings from their investigations. Students also learn to use counter-examples to disprove conjectures. Students can use *deductive reasoning* to assess the validity of conjectures and to formulate proofs.

### **Reflecting**

Good problem solvers regularly and consciously reflect on and monitor their own thought processes. By doing so, they are able to recognize when the technique they are using is not fruitful, and to make a conscious decision to switch to a different strategy, rethink the problem,

search for related content knowledge that may be helpful, and so forth. Students' problem solving skills are enhanced when they reflect on alternative ways to perform a task even if they have successfully completed it. Reflecting on the reasonableness of an answer by considering the original question or problem is another way in which students can improve their ability to make sense of problems.

### **Selecting Tools and Computational Strategies**

Students need to develop the ability to select the appropriate electronic tools, manipulatives, and computational strategies to perform particular mathematical tasks, to investigate mathematical ideas, and to solve problems.

**Calculators, Computers, Communications Technology.** Various types of technology are useful in learning and doing mathematics. Students can use calculators and computers to extend their capacity to investigate and analyse mathematical concepts and to reduce the time they might otherwise spend on purely mechanical activities.

Students can use calculators and computers to perform operations, make graphs, manipulate algebraic expressions, and organize and display data that are lengthier or more complex than those addressed in curriculum expectations suited to a paper-and-pencil approach. Students can also use calculators and computers in various ways to investigate number and graphing patterns, geometric relationships, and different representations; to simulate situations; and to extend problem solving. When students use calculators and computers in mathematics, they need to know when it is appropriate to apply their mental computation, reasoning, and estimation skills to predict results and check answers.

The computer and the calculator must be seen as important problem-solving tools to be used for many purposes. Computers and calculators are tools of mathematicians, and students should be given opportunities to select and use the particular applications that may be helpful to them as they search for their own solutions to problems.

Students may not be familiar with the use of some of the technologies suggested in the curriculum. When this is the case, it is important that teachers introduce their use in ways that build students' confidence and contribute to their understanding of the concepts being investigated. Students also need to understand the situations in which the new technology would be an appropriate choice of tool. Students' use of the tools should not be laborious or restricted to inputting and learning algorithmic steps. For example, when using spreadsheets and statistical software (e.g., *Fathom*), teachers could supply students with prepared data sets, and when using dynamic geometry software (e.g., *The Geometer's Sketchpad*), they could use pre-made sketches so that students' work with the software would be focused on manipulation of the data or the sketch, not on the inputting of data or the designing of the sketch.

Computer software programs can help students to collect, organize, and sort the data they gather, and to write, edit, and present reports on their findings. Whenever appropriate, students should be encouraged to select and use the communications technology that would best support and communicate their learning. Students, working individually or in groups, can use CD-ROMs and/or Internet websites to gain access to Statistics Canada, mathematics organizations, and other valuable sources of mathematical information around the world.

**Manipulatives.**<sup>2</sup> Students should be encouraged to select and use concrete learning tools to make models of mathematical ideas. Students need to understand that making their own models is a powerful means of building understanding and explaining their thinking to others. Using manipulatives to construct representations helps students to:

- see patterns and relationships;
- make connections between the concrete and the abstract;
- test, revise, and confirm their reasoning;
- remember how they solved a problem;
- communicate their reasoning to others.

**Computational Strategies.** Problem solving often requires students to select an appropriate computational strategy. They may need to apply a standard algorithm or to use technology for computation. They may also need to select strategies related to mental computation and estimation. Developing the ability to perform mental computation and to estimate is consequently an important aspect of student learning in mathematics.

Mental computation involves calculations done in the mind, with little or no use of paper and pencil. Students who have developed the ability to calculate mentally can select from and use a variety of procedures that take advantage of their knowledge and understanding of numbers, the operations, and their properties. Using their knowledge of the distributive property, for example, students can mentally compute 70% of 22 by first considering 70% of 20 and then adding 70% of 2. Used effectively, mental computation can encourage students to think more deeply about numbers and number relationships.

Knowing how to estimate, and knowing when it is useful to estimate and when it is necessary to have an exact answer, are important mathematical skills. Estimation is a useful tool for judging the reasonableness of a solution and for guiding students in their use of calculators. The ability to estimate depends on a well-developed sense of number and an understanding of place value. It can be a complex skill that requires decomposing numbers, compensating for errors, and perhaps even restructuring the problem. Estimation should not be taught as an isolated skill or a set of isolated rules and techniques. Knowing about calculations that are easy to perform and developing fluency in performing basic operations contribute to successful estimation.

## Connecting

Experiences that allow students to make connections – to see, for example, how concepts and skills from one strand of mathematics are related to those from another – will help them to grasp general mathematical principles. As they continue to make such connections, students begin to see that mathematics is more than a series of isolated skills and concepts and that they can use their learning in one area of mathematics to understand another. Seeing the relationships among procedures and concepts also helps deepen students' mathematical understanding.

In addition, making connections between the mathematics they study and its applications in their everyday lives helps students see the usefulness and relevance of mathematics beyond the classroom.

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2. See the Teaching Approaches section, on page 23 of this document, for additional information about the use of manipulatives in mathematics instruction.



## Representing

In secondary school mathematics, representing mathematical ideas and modelling situations generally take the form of numeric, geometric, graphical, algebraic, pictorial, and concrete representation, as well as representation using dynamic software. Students should be able to go from one representation to another, recognize the connections between representations, and use the different representations appropriately and as needed to solve problems. Learning the various forms of representation helps students to understand mathematical concepts and relationships; communicate their thinking, arguments, and understandings; recognize connections among related mathematical concepts; and use mathematics to model and interpret mathematical, physical, and social phenomena. When students are able to represent concepts in various ways, they develop flexibility in their thinking about those concepts. They are not inclined to perceive any single representation as “the math”; rather, they understand that it is just one of many representations that help them understand a concept.

## Communicating

Communication is the process of expressing mathematical ideas and understandings orally, visually, and in writing, using numbers, symbols, pictures, graphs, diagrams, and words. Students communicate for various purposes and for different audiences, such as the teacher, a peer, a group of students, or the whole class. Communication is an essential process in learning mathematics. Through communication, students are able to reflect upon and to clarify ideas, relationships, and mathematical arguments.

The development of mathematical language and symbolism fosters students’ communication skills. Teachers need to be aware of the various opportunities that exist in the classroom for helping students to communicate. For example, teachers can:

- model proper use of symbols, vocabulary, and notations in oral and written form;
- expect correct use of mathematical symbols and conventions in student work;
- ensure that students are exposed to and use new mathematical vocabulary as it is introduced (e.g., as they gather and interpret information; by providing opportunities to read, question, and discuss);
- provide feedback to students on their use of terminology and conventions;
- ask clarifying and extending questions and encourage students to ask themselves similar kinds of questions;
- ask students open-ended questions relating to specific topics or information;
- model ways in which various kinds of questions can be answered.

Effective classroom communication requires a supportive and respectful environment that makes all members of the class comfortable when they speak and when they question, react to, and elaborate on the statements of their classmates and the teacher.

The ability to provide effective explanations, and the understanding and application of correct mathematical notation in the development and presentation of mathematical ideas and solutions, are key aspects of effective communication in mathematics.

# Assessment and Evaluation of Student Achievement

## Basic Considerations

The primary purpose of assessment and evaluation is to improve student learning. Information gathered through assessment helps teachers to determine students' strengths and weaknesses in their achievement of the curriculum expectations in each course. This information also serves to guide teachers in adapting curriculum and instructional approaches to students' needs and in assessing the overall effectiveness of programs and classroom practices.

Assessment is the process of gathering information from a variety of sources (including assignments, demonstrations, projects, performances, and tests) that accurately reflects how well a student is achieving the curriculum expectations in a course. As part of assessment, teachers provide students with descriptive feedback that guides their efforts towards improvement. Evaluation refers to the process of judging the quality of student work on the basis of established criteria, and assigning a value to represent that quality.

Assessment and evaluation will be based on the provincial curriculum expectations and the achievement levels outlined in this document.

In order to ensure that assessment and evaluation are valid and reliable, and that they lead to the improvement of student learning, teachers must use assessment and evaluation strategies that:

- address both what students learn and how well they learn;
- are based both on the categories of knowledge and skills and on the achievement level descriptions given in the achievement chart on pages 20–21;
- are varied in nature, administered over a period of time, and designed to provide opportunities for students to demonstrate the full range of their learning;
- are appropriate for the learning activities used, the purposes of instruction, and the needs and experiences of the students;
- are fair to all students;
- accommodate the needs of students with special education needs, consistent with the strategies outlined in their Individual Education Plan;
- accommodate the needs of students who are learning the language of instruction (English or French);
- ensure that each student is given clear directions for improvement;
- promote students' ability to assess their own learning and to set specific goals;
- include the use of samples of students' work that provide evidence of their achievement;
- are communicated clearly to students and parents at the beginning of the course or the school term and at other appropriate points throughout the school year.

All curriculum expectations must be accounted for in instruction, but evaluation focuses on students' achievement of the overall expectations. A student's achievement of the overall expectations is evaluated on the basis of his or her achievement of related specific expectations (including the process expectations). The overall expectations are broad in nature, and the

specific expectations define the particular content or scope of the knowledge and skills referred to in the overall expectations. Teachers will use their professional judgement to determine which specific expectations should be used to evaluate achievement of the overall expectations, and which ones will be covered in instruction and assessment (e.g., through direct observation) but not necessarily evaluated.

The characteristics given in the achievement chart (pages 20–21) for level 3 represent the “provincial standard” for achievement of the expectations in a course. A complete picture of overall achievement at level 3 in a course in mathematics can be constructed by reading from top to bottom in the shaded column of the achievement chart, headed “70–79% (Level 3)”. Parents of students achieving at level 3 can be confident that their children will be prepared for work in subsequent courses.

Level 1 identifies achievement that falls much below the provincial standard, while still reflecting a passing grade. Level 2 identifies achievement that approaches the standard. Level 4 identifies achievement that surpasses the standard. It should be noted that achievement at level 4 does not mean that the student has achieved expectations beyond those specified for a particular course. It indicates that the student has achieved all or almost all of the expectations for that course, and that he or she demonstrates the ability to use the specified knowledge and skills in more sophisticated ways than a student achieving at level 3.

The Ministry of Education provides teachers with materials that will assist them in improving their assessment methods and strategies and, hence, their assessment of student achievement. These materials include samples of student work (exemplars) that illustrate achievement at each of the four levels.

### **The Achievement Chart for Mathematics**

The achievement chart that follows identifies four categories of knowledge and skills in mathematics. The achievement chart is a standard province-wide guide to be used by teachers. It enables teachers to make judgements about student work that are based on clear performance standards and on a body of evidence collected over time.

The purpose of the achievement chart is to:

- provide a common framework that encompasses the curriculum expectations for all courses outlined in this document;
- guide the development of quality assessment tasks and tools (including rubrics);
- help teachers to plan instruction for learning;
- assist teachers in providing meaningful feedback to students;
- provide various categories and criteria with which to assess and evaluate student learning.

***Categories of knowledge and skills.*** The categories, defined by clear criteria, represent four broad areas of knowledge and skills within which the expectations for any given mathematics course are organized. The four categories should be considered as interrelated, reflecting the wholeness and interconnectedness of learning.

The categories of knowledge and skills are described as follows:

*Knowledge and Understanding.* Subject-specific content acquired in each course (knowledge), and the comprehension of its meaning and significance (understanding).

*Thinking.* The use of critical and creative thinking skills and/or processes,<sup>3</sup> as follows:

- planning skills (e.g., understanding the problem, making a plan for solving the problem)
- processing skills (e.g., carrying out a plan, looking back at the solution)
- critical/creative thinking processes (e.g., inquiry, problem solving)

*Communication.* The conveying of meaning through various oral, written, and visual forms (e.g., providing explanations of reasoning or justification of results orally or in writing; communicating mathematical ideas and solutions in writing, using numbers and algebraic symbols, and visually, using pictures, diagrams, charts, tables, graphs, and concrete materials).

*Application.* The use of knowledge and skills to make connections within and between various contexts.

Teachers will ensure that student work is assessed and/or evaluated in a balanced manner with respect to the four categories, and that achievement of particular expectations is considered within the appropriate categories.

**Criteria.** Within each category in the achievement chart, criteria are provided, which are subsets of the knowledge and skills that define each category. For example, in Knowledge and Understanding, the criteria are “knowledge of content (e.g., facts, terms, procedural skills, use of tools)” and “understanding of mathematical concepts”. The criteria identify the aspects of student performance that are assessed and/or evaluated, and serve as guides to what to look for.

**Descriptors.** A “descriptor” indicates the characteristic of the student’s performance, with respect to a particular criterion, on which assessment or evaluation is focused. In the achievement chart, *effectiveness* is the descriptor used for each criterion in the Thinking, Communication, and Application categories. What constitutes effectiveness in any given performance task will vary with the particular criterion being considered. Assessment of effectiveness may therefore focus on a quality such as appropriateness, clarity, accuracy, precision, logic, relevance, significance, fluency, flexibility, depth, or breadth, as appropriate for the particular criterion. For example, in the Thinking category, assessment of effectiveness might focus on the degree of relevance or depth apparent in an analysis; in the Communication category, on clarity of expression or logical organization of information and ideas; or in the Application category, on appropriateness or breadth in the making of connections. Similarly, in the Knowledge and Understanding category, assessment of knowledge might focus on accuracy, and assessment of understanding might focus on the depth of an explanation. Descriptors help teachers to focus their assessment and evaluation on specific knowledge and skills for each category and criterion, and help students to better understand exactly what is being assessed and evaluated.

**Qualifiers.** A specific “qualifier” is used to define each of the four levels of achievement – that is, *limited* for level 1, *some* for level 2, *considerable* for level 3, and *a high degree* or *thorough* for level 4. A qualifier is used along with a descriptor to produce a description of performance at a particular level. For example, the description of a student’s performance at level 3 with respect to the first criterion in the Thinking category would be: “the student uses planning skills with *considerable* effectiveness”.

The descriptions of the levels of achievement given in the chart should be used to identify the level at which the student has achieved the expectations. In all of their courses, students should be provided with numerous and varied opportunities to demonstrate the full extent of their achievement of the curriculum expectations, across all four categories of knowledge and skills.

3. See the footnote on page 20, pertaining to the *mathematical processes*.

## Achievement Chart – Mathematics, Grades 9–12

Categories	50–59% (Level 1)	60–69% (Level 2)	70–79% (Level 3)	80–100% (Level 4)
<b>Knowledge and Understanding</b> <i>Subject-specific content acquired in each course (knowledge), and the comprehension of its meaning and significance (understanding)</i>				
<b>The student:</b>				
Knowledge of content (e.g., facts, terms, procedural skills, use of tools)	– demonstrates limited knowledge of content	– demonstrates some knowledge of content	– demonstrates considerable knowledge of content	– demonstrates thorough knowledge of content
Understanding of mathematical concepts	– demonstrates limited understanding of concepts	– demonstrates some understanding of concepts	– demonstrates considerable understanding of concepts	– demonstrates thorough understanding of concepts
<b>Thinking</b> <i>The use of critical and creative thinking skills and/or processes*</i>				
<b>The student:</b>				
Use of planning skills – understanding the problem (e.g., formulating and interpreting the problem, making conjectures) – making a plan for solving the problem	– uses planning skills with limited effectiveness	– uses planning skills with some effectiveness	– uses planning skills with considerable effectiveness	– uses planning skills with a high degree of effectiveness
Use of processing skills – carrying out a plan (e.g., collecting data, questioning, testing, revising, modelling, solving, inferring, forming conclusions) – looking back at the solution (e.g., evaluating reasonableness, making convincing arguments, reasoning, justifying, proving, reflecting)	– uses processing skills with limited effectiveness	– uses processing skills with some effectiveness	– uses processing skills with considerable effectiveness	– uses processing skills with a high degree of effectiveness
Use of critical/creative thinking processes (e.g., problem solving, inquiry)	– uses critical/creative thinking processes with limited effectiveness	– uses critical/creative thinking processes with some effectiveness	– uses critical/creative thinking processes with considerable effectiveness	– uses critical/creative thinking processes with a high degree of effectiveness

\* The processing skills and critical/creative thinking processes in the Thinking category include some but not all aspects of the *mathematical processes* described on pages 12–16 of this document. Some aspects of the mathematical processes relate to the other categories of the achievement chart.

Categories	50–59% (Level 1)	60–69% (Level 2)	70–79% (Level 3)	80–100% (Level 4)
<b>Communication</b> <i>The conveying of meaning through various forms</i>				
<b>The student:</b>				
Expression and organization of ideas and mathematical thinking (e.g., clarity of expression, logical organization), using oral, visual, and written forms (e.g., pictorial, graphic, dynamic, numeric, algebraic forms; concrete materials)	– expresses and organizes mathematical thinking with limited effectiveness	– expresses and organizes mathematical thinking with some effectiveness	– expresses and organizes mathematical thinking with considerable effectiveness	– expresses and organizes mathematical thinking with a high degree of effectiveness
Communication for different audiences (e.g., peers, teachers) and purposes (e.g., to present data, justify a solution, express a mathematical argument) in oral, visual, and written forms	– communicates for different audiences and purposes with limited effectiveness	– communicates for different audiences and purposes with some effectiveness	– communicates for different audiences and purposes with considerable effectiveness	– communicates for different audiences and purposes with a high degree of effectiveness
Use of conventions, vocabulary, and terminology of the discipline (e.g., terms, symbols) in oral, visual, and written forms	– uses conventions, vocabulary, and terminology of the discipline with limited effectiveness	– uses conventions, vocabulary, and terminology of the discipline with some effectiveness	– uses conventions, vocabulary, and terminology of the discipline with considerable effectiveness	– uses conventions, vocabulary, and terminology of the discipline with a high degree of effectiveness
<b>Application</b> <i>The use of knowledge and skills to make connections within and between various contexts</i>				
<b>The student:</b>				
Application of knowledge and skills in familiar contexts	– applies knowledge and skills in familiar contexts with limited effectiveness	– applies knowledge and skills in familiar contexts with some effectiveness	– applies knowledge and skills in familiar contexts with considerable effectiveness	– applies knowledge and skills in familiar contexts with a high degree of effectiveness
Transfer of knowledge and skills to new contexts	– transfers knowledge and skills to new contexts with limited effectiveness	– transfers knowledge and skills to new contexts with some effectiveness	– transfers knowledge and skills to new contexts with considerable effectiveness	– transfers knowledge and skills to new contexts with a high degree of effectiveness
Making connections within and between various contexts (e.g., connections between concepts, representations, and forms within mathematics; connections involving use of prior knowledge and experience; connections between mathematics, other disciplines, and the real world)	– makes connections within and between various contexts with limited effectiveness	– makes connections within and between various contexts with some effectiveness	– makes connections within and between various contexts with considerable effectiveness	– makes connections within and between various contexts with a high degree of effectiveness

Note: A student whose achievement is below 50% at the end of a course will not obtain a credit for the course.

## **Evaluation and Reporting of Student Achievement**

Student achievement must be communicated formally to students and parents by means of the Provincial Report Card, Grades 9–12. The report card provides a record of the student's achievement of the curriculum expectations in every course, at particular points in the school year or semester, in the form of a percentage grade. The percentage grade represents the quality of the student's overall achievement of the expectations for the course and reflects the corresponding level of achievement as described in the achievement chart for the discipline.

A final grade is recorded for every course, and a credit is granted and recorded for every course in which the student's grade is 50% or higher. The final grade for each course in Grades 9–12 will be determined as follows:

- Seventy per cent of the grade will be based on evaluations conducted throughout the course. This portion of the grade should reflect the student's most consistent level of achievement throughout the course, although special consideration should be given to more recent evidence of achievement.
- Thirty per cent of the grade will be based on a final evaluation in the form of an examination, performance, essay, and/or other method of evaluation suitable to the course content and administered towards the end of the course.

## **Reporting on Demonstrated Learning Skills**

The report card provides a record of the learning skills demonstrated by the student in every course, in the following five categories: Works Independently, Teamwork, Organization, Work Habits, and Initiative. The learning skills are evaluated using a four-point scale (E-Excellent, G-Good, S-Satisfactory, N-Needs Improvement). The separate evaluation and reporting of the learning skills in these five areas reflects their critical role in students' achievement of the curriculum expectations. To the extent possible, the evaluation of learning skills, apart from any that may be included as part of a curriculum expectation in a course, should not be considered in the determination of percentage grades.

# Some Considerations for Program Planning in Mathematics

Teachers who are planning a program in mathematics must take into account considerations in a number of important areas, including those discussed below.

## Teaching Approaches

To make new learning more accessible to students, teachers draw upon the knowledge and skills students have acquired in previous years – in other words, they help activate prior knowledge. It is important to assess where students are in their mathematical growth and to bring them forward in their learning.

In order to apply their knowledge effectively and to continue to learn, students must have a solid conceptual foundation in mathematics. Successful classroom practices involve students in activities that require higher-order thinking, with an emphasis on problem solving.<sup>4</sup> Students who have completed the elementary program should have a good grounding in the investigative approach to learning new concepts, including inquiry models of problem solving, and this approach continues to be important in the Grade 11 program.

Students in a mathematics class typically demonstrate diversity in the ways they learn best. It is important, therefore, that students have opportunities to learn in a variety of ways – individually, cooperatively, independently, with teacher direction, through hands-on experience, and through examples followed by practice. In mathematics, students are required to learn concepts, procedures, and processes and to acquire skills, and they become competent in these various areas with the aid of the instructional and learning strategies best suited to the particular type of learning. The approaches and strategies used in the classroom to help students meet the expectations of this curriculum will vary according to the object of the learning and the needs of the students.

Even at the secondary level, manipulatives are necessary tools for supporting the effective learning of mathematics. These concrete learning tools invite students to explore and represent abstract mathematical ideas in varied, concrete, tactile, and visually rich ways. Manipulatives are also a valuable aid to teachers. By analysing students' concrete representations of mathematical concepts and listening carefully to their reasoning, teachers can gain useful insights into students' thinking and provide supports to help enhance their thinking.<sup>5</sup>

All learning, especially new learning, should be embedded in well-chosen contexts for learning – that is, contexts that are broad enough to allow students to investigate initial understandings, identify and develop relevant supporting skills, and gain experience with varied and interesting applications of the new knowledge. Such rich contexts for learning open the door for students to see the “big ideas” of mathematics – that is, the major underlying principles, such as pattern or relationship. This understanding of key principles will enable and encourage students to use mathematical reasoning throughout their lives.

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4. See the resource document *Targeted Implementation & Planning Supports (TIPS4RM): Grade 7, 8, 9 Applied and 10 Applied Mathematics* (Toronto: Queen's Printer for Ontario, 2005) for helpful information about problem solving.

5. A list of manipulatives appropriate for use in intermediate and senior mathematics classrooms is provided in *Leading Math Success*, pages 48–49.



**Promoting Positive Attitudes Towards Learning Mathematics.** Students' attitudes have a significant effect on how students approach problem solving and how well they succeed in mathematics. Students who enjoy mathematics tend to perform well in their mathematics course work and are more likely to enrol in the more advanced mathematics courses.

Students develop positive attitudes when they are engaged in making mathematical conjectures, when they experience breakthroughs as they solve problems, when they see connections between important ideas, and when they observe an enthusiasm for mathematics on the part of their teachers.<sup>6</sup> With a positive attitude towards mathematics, students are able to make more sense of the mathematics they are working on, and to view themselves as effective learners of mathematics. They are also more likely to perceive mathematics as both useful and worthwhile, and to develop the belief that steady effort in learning mathematics pays off.

It is common for people to think that if they cannot solve problems quickly and easily, or in the right way, they must be inadequate. To gain confidence, students need to recognize that, for some mathematics problems, there may be several ways to arrive at a solution. They also need to understand that problem solving of almost any kind often requires a considerable expenditure of time and energy and a good deal of perseverance. They need to believe that they are capable of finding solutions, to counteract the frustration they may feel when they are not making progress towards solving a problem. Teachers can encourage students to develop a willingness to persist, to investigate, to reason, to explore alternative solutions, and to take the risks necessary to become successful problem solvers. They can help students develop confidence and reduce anxiety and frustration by providing them with problems that are challenging but not beyond their ability to solve. Problems at an appropriate level of challenge help students to learn while establishing a norm of perseverance for successful problem solving.

Collaborative learning enhances students' understanding of mathematics. Working cooperatively in groups reduces isolation and provides students with opportunities to share ideas and communicate their thinking in a supportive environment as they work together towards a common goal. Communication and the connections among ideas that emerge as students interact with one another enhance the quality of student learning.<sup>7</sup>

### **Planning Mathematics Programs for Students With Special Education Needs**

In planning mathematics courses for students with special education needs, teachers should begin by examining both the curriculum expectations for the course and the needs of the individual student to determine which of the following options is appropriate for the student:

- no accommodations<sup>8</sup> or modifications; or
- accommodations only; or
- modified expectations, with the possibility of accommodations.

If the student requires either accommodations or modified expectations, or both, the relevant information, as described in the following paragraphs, must be recorded in his or her Individual Education Plan (IEP). For a detailed discussion of the ministry's requirements for IEPs,

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6. *Leading Math Success*, p. 42.

7. *Leading Math Success*, p. 42.

8. "Accommodations" refers to individualized teaching and assessment strategies, human supports, and/or individualized equipment.

see *Individual Education Plans: Standards for Development, Program Planning, and Implementation, 2000* (referred to hereafter as *IEP Standards, 2000*). More detailed information about planning programs for students with special education needs can be found in *The Individual Education Plan (IEP): A Resource Guide, 2004* (referred to hereafter as *IEP Resource Guide, 2004*). (Both documents are available at <http://www.edu.gov.on.ca>.)

***Students Requiring Accommodations Only.*** With the aid of accommodations alone, some students are able to participate in the regular course curriculum and to demonstrate learning independently. (Accommodations do not alter the provincial curriculum expectations for the course.) The accommodations required to facilitate the student's learning must be identified in his or her IEP (see *IEP Standards, 2000*, page 11). A student's IEP is likely to reflect the same accommodations for many, or all, courses.

There are three types of accommodations. *Instructional accommodations* are changes in teaching strategies, including styles of presentation, methods of organization, or use of technology and multimedia. *Environmental accommodations* are changes that the student may require in the classroom and/or school environment, such as preferential seating or special lighting. *Assessment accommodations* are changes in assessment procedures that enable the student to demonstrate his or her learning, such as allowing additional time to complete tests or assignments or permitting oral responses to test questions (see *IEP Resource Guide, 2004*, page 29, for more examples).

If a student requires "accommodations only" in mathematics courses, assessment and evaluation of his or her achievement will be based on the appropriate course curriculum expectations and the achievement levels outlined in this document. The IEP box on the Provincial Report Card will not be checked, and no information on the provision of accommodations will be included.

***Students Requiring Modified Expectations.*** Some students will require modified expectations, which differ from the regular course expectations. For most students, modified expectations will be based on the regular course curriculum, with changes in the number and/or complexity of the expectations. It is important to monitor, and to reflect clearly in the student's IEP, the extent to which expectations have been modified. As noted in Section 7.12 of the ministry's policy document *Ontario Secondary Schools, Grades 9 to 12: Program and Diploma Requirements, 1999*, the principal will determine whether achievement of the modified expectations constitutes successful completion of the course, and will decide whether the student is eligible to receive a credit for the course. This decision must be communicated to the parents and the student.

When a student is expected to achieve most of the curriculum expectations for the course, the modified expectations should identify how they differ from the course expectations. When modifications are so extensive that achievement of the learning expectations is not likely to result in a credit, the expectations should specify the precise requirements or tasks on which the student's performance will be evaluated and that will be used to generate the course mark recorded on the Provincial Report Card. Modified expectations indicate the knowledge and/or skills the student is expected to demonstrate and have assessed in each reporting period (*IEP Standards, 2000*, pages 10 and 11). Modified expectations represent specific, realistic, observable, and measurable achievements and describe specific knowledge and/or skills that the student can demonstrate independently, given the appropriate assessment accommodations. The student's learning expectations must be reviewed in relation to the student's progress at least once every reporting period, and must be updated as necessary (*IEP Standards, 2000*, page 11).

If a student requires modified expectations in mathematics courses, assessment and evaluation of his or her achievement will be based on the learning expectations identified in the IEP and on the achievement levels outlined in this document. If some of the student's learning expectations for a course are modified but the student is working towards a credit for the course, it is sufficient simply to check the IEP box on the Provincial Report Card. If, however, the student's learning expectations are modified to such an extent that the principal deems that a credit will not be granted for the course, the IEP box must be checked and the appropriate statement from the *Guide to the Provincial Report Card, Grades 9–12, 1999* (page 8) must be inserted. The teacher's comments should include relevant information on the student's demonstrated learning of the modified expectations, as well as next steps for the student's learning in the course.

### **Program Considerations for English Language Learners**

Young people whose first language is not English enter Ontario secondary schools with diverse linguistic and cultural backgrounds. Some English language learners may have experience of highly sophisticated educational systems, while others may have come from regions where access to formal schooling was limited. All of these students bring a rich array of background knowledge and experience to the classroom, and all teachers must share in the responsibility for their English-language development.

Teachers of mathematics must incorporate appropriate adaptations and strategies for instruction and assessment to facilitate the success of the English language learners in their classrooms. These adaptations and strategies include:

- modification of some or all of the course expectations so that they are challenging but attainable for the learner at his or her present level of English proficiency, given the necessary support from the teacher;
- use of a variety of instructional strategies (e.g., extensive use of visual cues, scaffolding, manipulatives, pictures, diagrams, graphic organizers; attention to clarity of instructions; modelling of preferred ways of working in mathematics; previewing of textbooks; pre-teaching of key vocabulary; peer tutoring; strategic use of students' first languages);
- use of a variety of learning resources (e.g., visual material, simplified text, bilingual dictionaries, materials that reflect cultural diversity);
- use of assessment accommodations (e.g., granting of extra time; simplification of language used in problems and instructions; use of oral interviews, tasks requiring completion of graphic organizers or cloze sentences, learning logs, portfolios, demonstrations, or visual representations instead of assessment tasks that depend heavily on proficiency in English).

When learning expectations in a mathematics course are modified for English language learners, the ESL or ELD box must be checked on the student's report card. (See the *Guide to the Provincial Report Card, Grades 9–12, 1999*.)

For further information on supporting English language learners, refer to *The Ontario Curriculum, Grades 9 to 12: English As a Second Language and English Literacy Development, 1999* and the resource guide *Many Roots, Many Voices: Supporting English Language Learners in Every Classroom* (Ministry of Education, 2005).

## **Antidiscrimination Education in Mathematics**

To ensure that all students in the province have an equal opportunity to achieve their full potential, the curriculum must be free from bias and all students must be provided with a safe and secure environment, characterized by respect for others, that allows them to participate fully and responsibly in the educational experience.

Learning activities and resources used to implement the curriculum should be inclusive in nature, reflecting the range of experiences of students with varying backgrounds, abilities, interests, and learning styles. They should enable students to become more sensitive to the diverse cultures and perceptions of others, including Aboriginal peoples. By discussing aspects of the history of mathematics, teachers can help make students aware of the various cultural groups that have contributed to the evolution of mathematics over the centuries. Finally, students need to recognize that ordinary people use mathematics in a variety of everyday contexts, both at work and in their daily lives.

Connecting mathematical ideas to real-world situations through learning activities can enhance students' appreciation of the role of mathematics in human affairs, in areas including health, science, and the environment. Students can be made aware of the use of mathematics in contexts such as sampling and surveying and the use of statistics to analyse trends. Recognizing the importance of mathematics in such areas helps motivate students to learn and also provides a foundation for informed, responsible citizenship.

Teachers should have high expectations for all students. To achieve their mathematical potential, however, different students may need different kinds of support. Some boys, for example, may need additional support in developing their literacy skills in order to complete mathematical tasks effectively. For some girls, additional encouragement to envision themselves in careers involving mathematics may be beneficial. For example, teachers might consider providing strong role models in the form of female guest speakers who are mathematicians or who use mathematics in their careers.

## **Literacy and Inquiry/Research Skills**

Literacy skills can play an important role in student success in mathematics courses. Many of the activities and tasks students undertake in mathematics courses involve the use of written, oral, and visual communication skills. For example, students use language to record their observations, to explain their reasoning when solving problems, to describe their inquiries in both informal and formal contexts, and to justify their results in small-group conversations, oral presentations, and written reports. The language of mathematics includes special terminology. The study of mathematics consequently encourages students to use language with greater care and precision and enhances their ability to communicate effectively.

The Ministry of Education has facilitated the development of materials to support literacy instruction across the curriculum. Helpful advice for integrating literacy instruction in mathematics courses may be found in the following resource documents:

- *Think Literacy: Cross-Curricular Approaches, Grades 7–12, 2003*
- *Think Literacy: Cross-Curricular Approaches, Grades 7–12 – Mathematics: Subject-Specific Examples, Grades 10–12, 2005*

In all courses in mathematics, students will develop their ability to ask questions and to plan investigations to answer those questions and to solve related problems. Students need to learn a variety of research methods and inquiry approaches in order to carry out these investigations and to solve problems, and they need to be able to select the methods that are most appropriate for a particular inquiry. Students learn how to locate relevant information from a variety of sources, such as statistical databases, newspapers, and reports. As they advance through the grades, students will be expected to use such sources with increasing sophistication. They will also be expected to distinguish between primary and secondary sources, to determine their validity and relevance, and to use them in appropriate ways.

### **The Role of Technology in Mathematics**

Information and communication technologies (ICT) provide a range of tools that can significantly extend and enrich teachers' instructional strategies and support students' learning in mathematics. Teachers can use ICT tools and resources both for whole-class instruction and to design programs that meet diverse student needs. Technology can help to reduce the time spent on routine mathematical tasks and to allow students to devote more of their efforts to thinking and concept development. Useful ICT tools include simulations, multimedia resources, databases, sites that give access to large amounts of statistical data, and computer-assisted learning modules.

Applications such as databases, spreadsheets, dynamic geometry software, dynamic statistical software, graphing software, computer algebra systems (CAS), word-processing software, and presentation software can be used to support various methods of inquiry in mathematics. Technology also makes possible simulations of complex systems that can be useful for problem-solving purposes or when field studies on a particular topic are not feasible.

Information and communications technologies can be used in the classroom to connect students to other schools, at home and abroad, and to bring the global community into the local classroom.

Although the Internet is a powerful electronic learning tool, all students must be made aware of issues of privacy, safety, and responsible use, as well as of the ways in which the Internet can be used to promote hatred.

### **Career Education in Mathematics**

Teachers can promote students' awareness of careers involving mathematics by exploring applications of concepts and providing opportunities for career-related project work. Such activities allow students the opportunity to investigate mathematics-related careers compatible with their interests, aspirations, and abilities.

Students should be made aware that mathematical literacy and problem solving are valuable assets in an ever-widening range of jobs and careers in today's society. The knowledge and skills students acquire in mathematics courses are useful in fields such as science, business, engineering, and computer studies; in the hospitality, recreation, and tourism industries; and in the technical trades.

## **The Ontario Skills Passport and Essential Skills**

Teachers planning programs in mathematics need to be aware of the purpose and benefits of the Ontario Skills Passport (OSP). The OSP is a bilingual web-based resource that enhances the relevancy of classroom learning for students and strengthens school-work connections. The OSP provides clear descriptions of essential skills such as reading, writing, use of computers, measurement and calculation, and problem solving and includes an extensive database of occupation-specific workplace tasks that illustrate how workers use these skills on the job. The essential skills are transferable, in that they are used in virtually all occupations. The OSP also includes descriptions of important work habits, such as working safely, being reliable, and providing excellent customer service. The OSP is designed to help employers assess and record students' demonstration of these skills and work habits during their cooperative-education placements. Students can use the OSP to identify the skills and work habits they already have, plan further skill development, and show employers what they can do.

The skills described in the OSP are the essential skills that the Government of Canada and other national and international agencies have identified and validated, through extensive research, as the skills needed for work, learning, and life. Essential skills provide the foundation for learning all other skills and enable people to evolve with their jobs and adapt to workplace change. For information on the OSP and essential skills, visit: <http://skills.edu.gov.on.ca>.

## **Cooperative Education and Other Forms of Experiential Learning**

Cooperative education and other workplace experiences, such as job shadowing, field trips, and work experience, enable students to apply the skills they have developed in the classroom to real-life activities. Cooperative education and other workplace experiences also help to broaden students' knowledge of employment opportunities in a wide range of fields, including science and technology, research in the social sciences and humanities, and many forms of business administration. In addition, students develop their understanding of workplace practices, certifications, and the nature of employer-employee relationships. Teachers of mathematics should maintain links with colleges, universities, and community-based businesses to ensure students have access to hands-on experiences that will reinforce the knowledge and skills they have gained in school.

All cooperative education and other workplace experience will be provided in accordance with the ministry's policy document entitled *Cooperative Education and Other Forms of Experiential Learning: Policies and Procedures for Ontario Secondary Schools, 2000*.

## **Planning Program Pathways and Programs Leading to a Specialist High-Skills Major**

Mathematics courses are well suited for inclusion in programs leading to a Specialist High-Skills Major (SHSM) or in programs designed to provide pathways to particular apprenticeship or workplace destinations. In an SHSM program, mathematics courses can be bundled with other courses to provide the academic knowledge and skills important to particular industry sectors and required for success in the workplace and postsecondary education, including apprenticeship. Mathematics courses may also be combined with cooperative education credits to provide the workplace experience required for SHSM programs and for various program pathways to apprenticeship and workplace destinations. (SHSM programs would also include sector-specific learning opportunities offered by employers, skills-training centres, colleges, and community organizations.)

## Health and Safety in Mathematics

Although health and safety issues are not normally associated with mathematics, they may be important when learning involves fieldwork or investigations based on experimentation. Out-of-school fieldwork can provide an exciting and authentic dimension to students' learning experiences. It also takes the teacher and students out of the predictable classroom environment and into unfamiliar settings.

Teachers must preview and plan activities and expeditions carefully to protect students' health and safety. Teachers who provide support for students in workplace learning placements need to assess placements for safety and ensure students understand the importance of issues relating to health and safety in the workplace. Before taking part in workplace learning experiences, students must acquire the knowledge and skills needed for safe participation. Students must understand their rights to privacy and confidentiality as outlined in the Freedom of Information and Protection of Privacy Act. They have the right to function in an environment free from abuse and harassment, and they need to be aware of harassment and abuse issues in establishing boundaries for their own personal safety. They should be informed about school and community resources and school policies and reporting procedures with regard to all forms of abuse and harassment.

Policy/Program Memorandum No. 76A, "Workplace Safety and Insurance Coverage for Students in Work Education Programs" (September 2000), outlines procedures for ensuring the provision of Health and Safety Insurance Board coverage for students who are at least 14 years of age and are on placements of more than one day. (A one-day job shadowing or job twinning experience is treated as a field trip.) Teachers should also be aware of the minimum age requirements outlined in the Occupational Health and Safety Act for persons to be in or to be working in specific workplace settings. Relevant ministry policies are outlined in *Cooperative Education and Other Forms of Experiential Learning: Policies and Procedures for Ontario Secondary Schools, 2000*.

# Functions, Grade 11, University Preparation

(MCR3U)

This course introduces the mathematical concept of the function by extending students' experiences with linear and quadratic relations. Students will investigate properties of discrete and continuous functions, including trigonometric and exponential functions; represent functions numerically, algebraically, and graphically; solve problems involving applications of functions; investigate inverse functions; and develop facility in determining equivalent algebraic expressions. Students will reason mathematically and communicate their thinking as they solve multi-step problems.

**Prerequisite:** Principles of Mathematics, Grade 10, Academic

**Mathematical process expectations.** The mathematical processes are to be integrated into student learning in all areas of this course.

**Throughout this course, students will:**

- PROBLEM SOLVING**
  - develop, select, apply, compare, and adapt a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;
- REASONING AND PROVING**
  - develop and apply reasoning skills (e.g., use of inductive reasoning, deductive reasoning, and counter-examples; construction of proofs) to make mathematical conjectures, assess conjectures, and justify conclusions, and plan and construct organized mathematical arguments;
- REFLECTING**
  - demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by assessing the effectiveness of strategies and processes used, by proposing alternative approaches, by judging the reasonableness of results, by verifying solutions);
- SELECTING TOOLS AND COMPUTATIONAL STRATEGIES**
  - select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;
- CONNECTING**
  - make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, current events, art and culture, sports);
- REPRESENTING**
  - create a variety of representations of mathematical ideas (e.g., numeric, geometric, algebraic, graphical, pictorial representations; onscreen dynamic representations), connect and compare them, and select and apply the appropriate representations to solve problems;
- COMMUNICATING**
  - communicate mathematical thinking orally, visually, and in writing, using precise mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.



## Characteristics of Functions

### Overall Expectations

By the end of this course, students will:

- demonstrate an understanding of functions, their representations, and their inverses, and make connections between the algebraic and graphical representations of functions using transformations;
- determine the zeros and the maximum or minimum of a quadratic function, and solve problems involving quadratic functions, including those arising from real-world applications;
- demonstrate an understanding of equivalence as it relates to simplifying polynomial, radical, and rational expressions.

### Specific Expectations

#### Representing Functions

By the end of this course, students will:

- explain the meaning of the term *function*, and distinguish a function from a relation that is not a function, through investigation of linear and quadratic relations using a variety of representations (i.e., tables of values, mapping diagrams, graphs, function machines, equations) and strategies (e.g., identifying a one-to-one or many-to-one mapping; using the vertical line test) (**Sample problem:** Investigate, using numeric and graphical representations, whether the relation  $x = y^2$  is a function, and justify your reasoning.);
- represent linear and quadratic functions using function notation, given their equations, tables of values, or graphs, and substitute into and evaluate functions [e.g., evaluate  $f\left(\frac{1}{2}\right)$ , given  $f(x) = 2x^2 + 3x - 1$ ];
- explain the meanings of the terms *domain* and *range*, through investigation using numeric, graphical, and algebraic representations of the functions  $f(x) = x$ ,  $f(x) = x^2$ ,  $f(x) = \sqrt{x}$ , and  $f(x) = \frac{1}{x}$ ; describe the domain and range of a function appropriately (e.g., for  $y = x^2 + 1$ , the domain is the set of real numbers, and the range is  $y \geq 1$ ); and explain any restrictions on the domain and range in contexts arising from real-world applications (**Sample problem:** A quadratic function represents the relationship between the height of a ball and the time elapsed since the ball was thrown. What physical factors will restrict the domain and range of the quadratic function?);
- relate the process of determining the inverse of a function to their understanding of reverse processes (e.g., applying inverse operations);
- determine the numeric or graphical representation of the inverse of a linear or quadratic function, given the numeric, graphical, or algebraic representation of the function, and make connections, through investigation using a variety of tools (e.g., graphing technology, Mira, tracing paper), between the graph of a function and the graph of its inverse (e.g., the graph of the inverse is the reflection of the graph of the function in the line  $y = x$ ) (**Sample problem:** Given a graph and a table of values representing population over time, produce a table of values for the inverse and graph the inverse on a new set of axes.);
- determine, through investigation, the relationship between the domain and range of a function and the domain and range of the inverse relation, and determine

- whether or not the inverse relation is a function [**Sample problem:** Given the graph of  $f(x) = x^2$ , graph the inverse relation. Compare the domain and range of the function with the domain and range of the inverse relation, and investigate connections to the domain and range of the functions  $g(x) = \sqrt{x}$ , and  $h(x) = -\sqrt{x}$ .];
- determine, using function notation when appropriate, the algebraic representation of the inverse of a linear or quadratic function, given the algebraic representation of the function [e.g.,  $f(x) = (x - 2)^2 - 5$ ], and make connections, through investigation using a variety of tools (e.g., graphing technology, Mira, tracing paper), between the algebraic representations of a function and its inverse (e.g., the inverse of a linear function involves applying the inverse operations in the reverse order) (**Sample problem:** Given the equations of several linear functions, graph the functions and their inverses, determine the equations of the inverses, and look for patterns that connect the equation of each linear function with the equation of the inverse.);
  - determine, through investigation using technology, and describe the roles of the parameters  $a$ ,  $k$ ,  $d$ , and  $c$  in functions of the form  $y = af(k(x - d)) + c$  in terms of transformations on the graphs of  $f(x) = x$ ,  $f(x) = x^2$ ,  $f(x) = \sqrt{x}$ , and  $f(x) = \frac{1}{x}$  (i.e., translations; reflections in the axes; vertical and horizontal stretches and compressions) [**Sample problem:** Investigate the graph  $f(x) = 3(x - d)^2 + 5$  for various values of  $d$ , using technology, and describe the effects of changing  $d$  in terms of a transformation.];
  - sketch graphs of  $y = af(k(x - d)) + c$  by applying one or more transformations to the graphs of  $f(x) = x$ ,  $f(x) = x^2$ ,  $f(x) = \sqrt{x}$ , and  $f(x) = \frac{1}{x}$ , and state the domain and range of the transformed functions [**Sample problem:** Transform the graph of  $f(x)$  to sketch  $g(x)$ , and state the domain and range of each function, for the following:  

$$f(x) = \sqrt{x}, g(x) = \sqrt{x - 4}; f(x) = \frac{1}{x},$$

$$g(x) = -\frac{1}{x + 1}.]$$
- ### Solving Problems Involving Quadratic Functions
- By the end of this course, students will:
- determine the number of zeros (i.e.,  $x$ -intercepts) of a quadratic function, using a variety of strategies (e.g., inspecting graphs; factoring; calculating the discriminant) (**Sample problem:** Investigate, using graphing technology and algebraic techniques, the transformations that affect the number of zeros for a given quadratic function.);
  - determine the maximum or minimum value of a quadratic function whose equation is given in the form  $f(x) = ax^2 + bx + c$ , using an algebraic method (e.g., completing the square; factoring to determine the zeros and averaging the zeros) [**Sample problem:** Explain how partially factoring  $f(x) = 3x^2 - 6x + 5$  into the form  $f(x) = 3x(x - 2) + 5$  helps you determine the minimum of the function.];
  - solve problems involving quadratic functions arising from real-world applications and represented using function notation [**Sample problem:** The profit,  $P(x)$ , of a video company, in thousands of dollars, is given by  $P(x) = -5x^2 + 550x - 5000$ , where  $x$  is the amount spent on advertising, in thousands of dollars. Determine the maximum profit that the company can make, and the amounts spent on advertising that will result in a profit and that will result in a profit of at least \$4 000 000.];

- determine, through investigation, the transformational relationship among the family of quadratic functions that have the same zeros, and determine the algebraic representation of a quadratic function, given the real roots of the corresponding quadratic equation and a point on the function [**Sample problem:** Determine the equation of the quadratic function that passes through (2, 5) if the roots of the corresponding quadratic equation are  $1 + \sqrt{5}$  and  $1 - \sqrt{5}$ .];
- solve problems involving the intersection of a linear function and a quadratic function graphically and algebraically (e.g., determining the time when two identical cylindrical water tanks contain equal volumes of water, if one tank is being filled at a constant rate and the other is being emptied through a hole in the bottom) [**Sample problem:** Determine, through investigation, the equations of the lines that have a slope of 2 and that intersect the quadratic function  $f(x) = x(6 - x)$  once; twice; never.].
- simplify rational expressions by adding, subtracting, multiplying, and dividing, and state the restrictions on the variable values (**Sample problem:** Simplify  $\frac{2x}{4x^2 + 6x} - \frac{3}{2x + 3}$ , and state the restrictions on the variable.);
- determine if two given algebraic expressions are equivalent (i.e., by simplifying; by substituting values) [**Sample problem:** Determine if the expressions  $\frac{2x^2 - 4x - 6}{x + 1}$  and  $8x^2 - 2x(4x - 1) - 6$  are equivalent.].

#### **Determining Equivalent Algebraic Expressions\***

By the end of this course, students will:

- simplify polynomial expressions by adding, subtracting, and multiplying (**Sample problem:** Write and simplify an expression for the volume of a cube with edge length  $2x + 1$ .);
- verify, through investigation with and without technology, that  $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ ,  $a \geq 0$ ,  $b \geq 0$ , and use this relationship to simplify radicals (e.g.,  $\sqrt{24}$ ) and radical expressions obtained by adding, subtracting, and multiplying [e.g.,  $(2 + \sqrt{6})(3 - \sqrt{12})$ ].

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\*The knowledge and skills described in the expectations in this section are to be introduced as needed and applied and consolidated in solving problems throughout the course.

# Exponential Functions

## Overall Expectations

By the end of this course, students will:

- evaluate powers with rational exponents, simplify expressions containing exponents, and describe properties of exponential functions represented in a variety of ways;
- make connections between the numeric, graphical, and algebraic representations of exponential functions;
- identify and represent exponential functions, and solve problems involving exponential functions, including those arising from real-world applications.

## Specific Expectations

### Representing Exponential Functions

By the end of this course, students will:

- graph, with and without technology, an exponential relation, given its equation in the form  $y = a^x$  ( $a > 0, a \neq 1$ ), define this relation as the function  $f(x) = a^x$ , and explain why it is a function;
- determine, through investigation using a variety of tools (e.g., calculator, paper and pencil, graphing technology) and strategies (e.g., patterning; finding values from a graph; interpreting the exponent laws), the value of a power with a rational exponent (i.e.,  $x^{\frac{m}{n}}$ , where  $x > 0$  and  $m$  and  $n$  are integers) (**Sample problem:** The exponent laws suggest that  $4^{\frac{1}{2}} \times 4^{\frac{1}{2}} = 4^1$ . What value would you assign to  $4^{\frac{1}{2}}$ ? What value would you assign to  $27^{\frac{1}{3}}$ ? Explain your reasoning. Extend your reasoning to make a generalization about the meaning of  $x^{\frac{1}{n}}$ , where  $x > 0$  and  $n$  is a natural number.);
- simplify algebraic expressions containing integer and rational exponents [e.g.,  $(x^3) \div (x^{\frac{1}{2}})$ ,  $(x^6 y^3)^{\frac{1}{3}}$ ], and evaluate numerical expressions containing integer and rational exponents and rational bases [e.g.,  $2^{-3}$ ,  $(-6)^3$ ,  $4^{\frac{1}{2}}$ ,  $1.01^{120}$ ];
- determine, through investigation, and describe key properties relating to domain and range, intercepts, increasing/decreasing

intervals, and asymptotes (e.g., the domain is the set of real numbers; the range is the set of positive real numbers; the function either increases or decreases throughout its domain) for exponential functions represented in a variety of ways [e.g., tables of values, mapping diagrams, graphs, equations of the form  $f(x) = a^x$  ( $a > 0, a \neq 1$ ), function machines] [**Sample problem:** Graph  $f(x) = 2^x$ ,  $g(x) = 3^x$ , and  $h(x) = 0.5^x$  on the same set of axes. Make comparisons between the graphs, and explain the relationship between the  $y$ -intercepts.]

### Connecting Graphs and Equations of Exponential Functions

By the end of this course, students will:

- distinguish exponential functions from linear and quadratic functions by making comparisons in a variety of ways (e.g., comparing rates of change using finite differences in tables of values; identifying a constant ratio in a table of values; inspecting graphs; comparing equations) [**Sample problem:** Explain in a variety of ways how you can distinguish the exponential function  $f(x) = 2^x$  from the quadratic function  $f(x) = x^2$  and the linear function  $f(x) = 2x$ .];
- determine, through investigation using technology, and describe the roles of the parameters  $a$ ,  $k$ ,  $d$ , and  $c$  in functions of the form  $y = af(k(x - d)) + c$  in terms of

- transformations on the graph of  $f(x) = a^x$  ( $a > 0, a \neq 1$ ) (i.e., translations; reflections in the axes; vertical and horizontal stretches and compressions) [**Sample problem:** Investigate the graph  $f(x) = 3^{x-d} - 5$  for various values of  $d$ , using technology, and describe the effects of changing  $d$  in terms of a transformation.];
- sketch graphs of  $y = af(k(x-d)) + c$  by applying one or more transformations to the graph of  $f(x) = a^x, a > 0, a \neq 1$ , and state the domain and range of the transformed functions [**Sample problem:** Transform the graph of  $f(x) = 3^x$  to sketch  $g(x) = 3^{-(x+1)} - 2$ , and state the domain and range of each function.];
  - determine, through investigation using technology, that the equation of a given exponential function can be expressed using different bases [e.g.,  $f(x) = 9^x$  can be expressed as  $f(x) = 3^{2x}$ ], and explain the connections between the equivalent forms in a variety of ways (e.g., comparing graphs; using transformations; using the exponent laws);
  - represent an exponential function with an equation, given its graph or its properties (**Sample problem:** Write two equations to represent the same exponential function with a  $y$ -intercept of 5 and an asymptote at  $y = 3$ . Investigate whether other exponential functions have the same properties. Use transformations to explain your observations.);
- (e.g., concrete materials such as number cubes, coins; measurement tools such as electronic probes), or from secondary sources (e.g., websites such as Statistics Canada, E-STAT), and graph the data (**Sample problem:** Collect data and graph the cooling curve representing the relationship between temperature and time for hot water cooling in a porcelain mug. Predict the shape of the cooling curve when hot water cools in an insulated mug. Test your prediction.);
- identify exponential functions, including those that arise from real-world applications involving growth and decay (e.g., radioactive decay, population growth, cooling rates, pressure in a leaking tire), given various representations (i.e., tables of values, graphs, equations), and explain any restrictions that the context places on the domain and range (e.g., ambient temperature limits the range for a cooling curve) (**Sample problem:** Using data from Statistics Canada, investigate to determine if there was a period of time over which the increase in Canada's national debt could be modelled using an exponential function.);
  - solve problems using given graphs or equations of exponential functions arising from a variety of real-world applications (e.g., radioactive decay, population growth, height of a bouncing ball, compound interest) by interpreting the graphs or by substituting values for the exponent into the equations [**Sample problem:** The temperature of a cooling liquid over time can be modelled by the exponential function  $T(x) = 60\left(\frac{1}{2}\right)^{\frac{x}{30}} + 20$ , where  $T(x)$  is the temperature, in degrees Celsius, and  $x$  is the elapsed time, in minutes. Graph the function and determine how long it takes for the temperature to reach  $28^\circ\text{C}$ .].

### **Solving Problems Involving Exponential Functions**

By the end of this course, students will:

- collect data that can be modelled as an exponential function, through investigation with and without technology, from primary sources, using a variety of tools

## Discrete Functions

### Overall Expectations

By the end of this course, students will:

- demonstrate an understanding of recursive sequences, represent recursive sequences in a variety of ways, and make connections to Pascal's triangle;
- demonstrate an understanding of the relationships involved in arithmetic and geometric sequences and series, and solve related problems;
- make connections between sequences, series, and financial applications, and solve problems involving compound interest and ordinary annuities.

### Specific Expectations

#### Representing Sequences

By the end of this course, students will:

- make connections between sequences and discrete functions, represent sequences using function notation, and distinguish between a discrete function and a continuous function [e.g.,  $f(x) = 2x$ , where the domain is the set of natural numbers, is a discrete linear function, and its graph is a set of equally spaced points;  $f(x) = 2x$ , where the domain is the set of real numbers, is a continuous linear function and its graph is a straight line];
- determine and describe (e.g., in words; using flow charts) a recursive procedure for generating a sequence, given the initial terms (e.g., 1, 3, 6, 10, 15, 21, ...), and represent sequences as discrete functions in variety of ways (e.g., tables of values, graphs);
- connect the formula for the  $n$ th term of a sequence to the representation in function notation, and write terms of a sequence given one of these representations or a recursion formula;
- represent a sequence algebraically using a recursion formula, function notation, or the formula for the  $n$ th term [e.g., represent 2, 4, 8, 16, 32, 64, ... as  $t_1 = 2$ ;  $t_n = 2t_{n-1}$ , as  $f(n) = 2^n$ , or as  $t_n = 2^n$ , or represent  $\frac{1}{2}$ ,  $\frac{2}{3}$ ,  $\frac{3}{4}$ ,

$$\frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \dots \text{ as } t_1 = \frac{1}{2}; t_n = t_{n-1} + \frac{1}{n(n+1)},$$

$$\text{as } f(n) = \frac{n}{n+1}, \text{ or as } t_n = \frac{n}{n+1}, \text{ where } n$$

is a natural number], and describe the information that can be obtained by inspecting each representation (e.g., function notation or the formula for the  $n$ th term may show the type of function; a recursion formula shows the relationship between terms)

(**Sample Problem:** Represent the sequence 0, 3, 8, 15, 24, 35, ... using a recursion formula, function notation, and the formula for the  $n$ th term. Explain why this sequence can be described as a discrete quadratic function. Explore how to identify a sequence as a discrete quadratic function by inspecting the recursion formula.);

- determine, through investigation, recursive patterns in the Fibonacci sequence, in related sequences, and in Pascal's triangle, and represent the patterns in a variety of ways (e.g., tables of values, algebraic notation);
- determine, through investigation, and describe the relationship between Pascal's triangle and the expansion of binomials, and apply the relationship to expand binomials raised to whole-number exponents [e.g.,  $(1+x)^4$ ,  $(2x-1)^5$ ,  $(2x-y)^6$ ,  $(x^2+1)^5$ ].

### **Investigating Arithmetic and Geometric Sequences and Series**

By the end of this course, students will:

- identify sequences as arithmetic, geometric, or neither, given a numeric or algebraic representation;
- determine the formula for the general term of an arithmetic sequence [i.e.,  $t_n = a + (n - 1)d$ ] or geometric sequence (i.e.,  $t_n = ar^{n-1}$ ), through investigation using a variety of tools (e.g., linking cubes, algebra tiles, diagrams, calculators) and strategies (e.g., patterning; connecting the steps in a numerical example to the steps in the algebraic development), and apply the formula to calculate any term in a sequence;
- determine the formula for the sum of an arithmetic or geometric series, through investigation using a variety of tools (e.g., linking cubes, algebra tiles, diagrams, calculators) and strategies (e.g., patterning; connecting the steps in a numerical example to the steps in the algebraic development), and apply the formula to calculate the sum of a given number of consecutive terms (**Sample problem:** Given the array built with grey and white connecting cubes, investigate how different ways of determining the total number of grey cubes can be used to evaluate the sum of the arithmetic series  $1 + 2 + 3 + 4 + 5$ . Extend the series, use patterning to make generalizations for finding the sum, and test the generalizations for other arithmetic series.);


- solve problems involving arithmetic and geometric sequences and series, including those arising from real-world applications.

### **Solving Problems Involving Financial Applications**

By the end of this course, students will:

- make and describe connections between simple interest, arithmetic sequences, and linear growth, through investigation with technology (e.g., use a spreadsheet or graphing calculator to make simple interest calculations, determine first differences in the amounts over time, and graph amount versus time) [**Sample problem:** Describe an investment that could be represented by the function  $f(x) = 500(1.05x)$ .];
- make and describe connections between compound interest, geometric sequences, and exponential growth, through investigation with technology (e.g., use a spreadsheet to make compound interest calculations, determine finite differences in the amounts over time, and graph amount versus time) [**Sample problem:** Describe an investment that could be represented by the function  $f(x) = 500(1.05)^x$ .];
- solve problems, using a scientific calculator, that involve the calculation of the amount,  $A$  (also referred to as future value,  $FV$ ), the principal,  $P$  (also referred to as present value,  $PV$ ), or the interest rate per compounding period,  $i$ , using the compound interest formula in the form  $A = P(1 + i)^n$  [or  $FV = PV(1 + i)^n$ ] (**Sample problem:** Two investments are available, one at 6% compounded annually and the other at 6% compounded monthly. Investigate graphically the growth of each investment, and determine the interest earned from depositing \$1000 in each investment for 10 years.);

- determine, through investigation using technology (e.g., scientific calculator; the TVM solver in a graphing calculator; online tools), and describe strategies (e.g., guessing and checking; using the power of a power rule for exponents; using graphs) for calculating the number of compounding periods,  $n$ , using the compound interest formula in the form  $A = P(1 + i)^n$  [or  $FV = PV(1 + i)^n$ ], and solve related problems;
- explain the meaning of the term *annuity*, and determine the relationships between ordinary annuities, geometric series, and exponential growth, through investigation with technology in situations where the compounding period and the payment period are the same (e.g., use a spreadsheet to determine and graph the future value of an ordinary annuity for varying numbers of compounding periods; investigate how the contributions of each payment to the future value of an ordinary annuity are related to the terms of a geometric series);
- determine, through investigation using technology (e.g., the TVM Solver in a graphing calculator; online tools), the effects of changing the conditions (i.e., the payments, the frequency of the payments, the interest rate, the compounding period) of ordinary annuities in situations where the compounding period and the payment period are the same (e.g., long-term savings plans, loans) (**Sample problem:** Compare the amounts at age 65 that would result from making an annual deposit of \$1000 starting at age 20, or from making an annual deposit of \$3000 starting at age 50, to an RRSP that earns 6% interest per annum, compounded annually. What is the total of the deposits in each situation?);
- solve problems, using technology (e.g., scientific calculator, spreadsheet, graphing calculator), that involve the amount, the present value, and the regular payment of an ordinary annuity in situations where the compounding period and the payment period are the same (e.g., calculate the total interest paid over the life of a loan, using a spreadsheet, and compare the total interest with the original principal of the loan).



## Trigonometric Functions

### Overall Expectations

By the end of this course, students will:

- determine the values of the trigonometric ratios for angles less than  $360^\circ$ ; prove simple trigonometric identities; and solve problems using the primary trigonometric ratios, the sine law, and the cosine law;
- demonstrate an understanding of periodic relationships and sinusoidal functions, and make connections between the numeric, graphical, and algebraic representations of sinusoidal functions;
- identify and represent sinusoidal functions, and solve problems involving sinusoidal functions, including those arising from real-world applications.

### Specific Expectations

#### *Determining and Applying Trigonometric Ratios*

By the end of this course, students will:

- determine the exact values of the sine, cosine, and tangent of the special angles:  $0^\circ$ ,  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$ , and  $90^\circ$ ;
- determine the values of the sine, cosine, and tangent of angles from  $0^\circ$  to  $360^\circ$ , through investigation using a variety of tools (e.g., dynamic geometry software, graphing tools) and strategies (e.g., applying the unit circle; examining angles related to special angles);
- determine the measures of two angles from  $0^\circ$  to  $360^\circ$  for which the value of a given trigonometric ratio is the same;
- define the secant, cosecant, and cotangent ratios for angles in a right triangle in terms of the sides of the triangle (e.g.,  $\sec A = \frac{\text{hypotenuse}}{\text{adjacent}}$ ), and relate these ratios to the cosine, sine, and tangent ratios (e.g.,  $\sec A = \frac{1}{\cos A}$ );
- prove simple trigonometric identities, using the Pythagorean identity  $\sin^2 x + \cos^2 x = 1$ ; the quotient identity  $\tan x = \frac{\sin x}{\cos x}$ ; and the reciprocal identities  $\sec x = \frac{1}{\cos x}$ ,  $\csc x = \frac{1}{\sin x}$ , and  $\cot x = \frac{1}{\tan x}$

(*Sample problem:* Prove that  $1 - \cos^2 x = \sin x \cos x \tan x$ .);

- pose and solve problems involving right triangles and oblique triangles in two-dimensional settings, using the primary trigonometric ratios, the cosine law, and the sine law (including the ambiguous case);
- pose and solve problems involving right triangles and oblique triangles in three-dimensional settings, using the primary trigonometric ratios, the cosine law, and the sine law (*Sample problem:* Explain how a surveyor could find the height of a vertical cliff that is on the other side of a raging river, using a measuring tape, a theodolite, and some trigonometry. Create data that the surveyor might measure, and use the data to calculate the height of the cliff.).

#### *Connecting Graphs and Equations of Sinusoidal Functions*

By the end of this course, students will:

- describe key properties (e.g., cycle, amplitude, period) of periodic functions arising from real-world applications (e.g., natural gas consumption in Ontario, tides in the Bay of Fundy), given a numerical or graphical representation;

- predict, by extrapolating, the future behaviour of a relationship modelled using a numeric or graphical representation of a periodic function (e.g., predicting hours of daylight on a particular date from previous measurements; predicting natural-gas consumption in Ontario from previous consumption);
- make connections between the sine ratio and the sine function and between the cosine ratio and the cosine function by graphing the relationship between angles from  $0^\circ$  to  $360^\circ$  and the corresponding sine ratios or cosine ratios, with or without technology (e.g., by generating a table of values using a calculator; by unwrapping the unit circle), defining this relationship as the function  $f(x) = \sin x$  or  $f(x) = \cos x$ , and explaining why the relationship is a function;
- sketch the graphs of  $f(x) = \sin x$  and  $f(x) = \cos x$  for angle measures expressed in degrees, and determine and describe key properties (i.e., cycle, domain, range, intercepts, amplitude, period, maximum and minimum values, increasing/decreasing intervals);
- determine, through investigation using technology, and describe the roles of the parameters  $a$ ,  $k$ ,  $d$ , and  $c$  in functions of the form  $y = af(k(x - d)) + c$  in terms of transformations on the graphs of  $f(x) = \sin x$  and  $f(x) = \cos x$  with angles expressed in degrees (i.e., translations; reflections in the axes; vertical and horizontal stretches and compressions) [**Sample problem:** Investigate the graph  $f(x) = 2\sin(x - d) + 10$  for various values of  $d$ , using technology, and describe the effects of changing  $d$  in terms of a transformation.];
- determine the amplitude, period, phase shift, domain, and range of sinusoidal functions whose equations are given in the form  $f(x) = a\sin(k(x - d)) + c$  or  $f(x) = a\cos(k(x - d)) + c$ ;
- sketch graphs of  $y = af(k(x - d)) + c$  by applying one or more transformations to the graphs of  $f(x) = \sin x$  and  $f(x) = \cos x$ , and state the domain and range of the transformed functions [**Sample problem:** Transform the graph of  $f(x) = \cos x$  to sketch  $g(x) = 3\cos 2x - 1$ , and state the domain and range of each function.];
- represent a sinusoidal function with an equation, given its graph or its properties [**Sample problem:** A sinusoidal function has an amplitude of 2 units, a period of  $180^\circ$ , and a maximum at  $(0, 3)$ . Represent the function with an equation in two different ways.].

### **Solving Problems Involving Sinusoidal Functions**

By the end of this course, students will:

- collect data that can be modelled as a sinusoidal function (e.g., voltage in an AC circuit, sound waves), through investigation with and without technology, from primary sources, using a variety of tools (e.g., concrete materials; measurement tools such as motion sensors), or from secondary sources (e.g., websites such as Statistics Canada, E-STAT), and graph the data (**Sample problem:** Measure and record distance–time data for a swinging pendulum, using a motion sensor or other measurement tools, and graph the data.);
- identify sinusoidal functions, including those that arise from real-world applications involving periodic phenomena, given various representations (i.e., tables of values, graphs, equations), and explain any restrictions that the context places on the domain and range (**Sample problem:** Using data from Statistics Canada, investigate to determine if there was a period of time over which changes in the population of Canadians aged 20–24 could be modelled using a sinusoidal function.);

- determine, through investigation, how sinusoidal functions can be used to model periodic phenomena that do not involve angles [**Sample problem:** Investigate, using graphing technology in degree mode, and explain how the function  $h(t) = 5 \sin(30(t + 3))$  approximately models the relationship between the height and the time of day for a tide with an amplitude of 5 m, if high tide is at midnight.];
- predict the effects on a mathematical model (i.e., graph, equation) of an application involving sinusoidal functions when the conditions in the application are varied (e.g., varying the conditions, such as speed and direction, when walking in a circle in front of a motion sensor) (**Sample problem:** The relationship between the height above the ground of a person riding a Ferris wheel and time can be modelled using a sinusoidal function. Describe the effect on this function if the platform from which the person enters the ride is raised by 1 m and if the Ferris wheel turns twice as fast.);
- pose and solve problems based on applications involving a sinusoidal function by using a given graph or a graph generated with technology from its equation [**Sample problem:** The height above the ground of a rider on a Ferris wheel can be modelled by the sine function  $h(t) = 25 \sin 3(t - 30) + 27$ , where  $h(t)$  is the height, in metres, and  $t$  is the time, in seconds. Graph the function, using graphing technology in degree mode, and determine the maximum and minimum heights of the rider, the height after 30 s, and the time required to complete one revolution.].

# Functions and Applications, Grade 11, University/College Preparation

(MCF3M)

This course introduces basic features of the function by extending students' experiences with quadratic relations. It focuses on quadratic, trigonometric, and exponential functions and their use in modelling real-world situations. Students will represent functions numerically, graphically, and algebraically; simplify expressions; solve equations; and solve problems relating to applications. Students will reason mathematically and communicate their thinking as they solve multi-step problems.

**Prerequisite:** Principles of Mathematics, Grade 10, Academic, or Foundations of Mathematics, Grade 10, Applied

**Mathematical process expectations.** The mathematical processes are to be integrated into student learning in all areas of this course.

### Throughout this course, students will:

- PROBLEM SOLVING**
  - develop, select, apply, compare, and adapt a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;
- REASONING AND PROVING**
  - develop and apply reasoning skills (e.g., use of inductive reasoning, deductive reasoning, and counter-examples; construction of proofs) to make mathematical conjectures, assess conjectures, and justify conclusions, and plan and construct organized mathematical arguments;
- REFLECTING**
  - demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by assessing the effectiveness of strategies and processes used, by proposing alternative approaches, by judging the reasonableness of results, by verifying solutions);
- SELECTING TOOLS AND COMPUTATIONAL STRATEGIES**
  - select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;
- CONNECTING**
  - make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, current events, art and culture, sports);
- REPRESENTING**
  - create a variety of representations of mathematical ideas (e.g., numeric, geometric, algebraic, graphical, pictorial representations; onscreen dynamic representations), connect and compare them, and select and apply the appropriate representations to solve problems;
- COMMUNICATING**
  - communicate mathematical thinking orally, visually, and in writing, using precise mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.

# Quadratic Functions

## Overall Expectations

By the end of this course, students will:

- expand and simplify quadratic expressions, solve quadratic equations, and relate the roots of a quadratic equation to the corresponding graph;
- demonstrate an understanding of functions, and make connections between the numeric, graphical, and algebraic representations of quadratic functions;
- solve problems involving quadratic functions, including those arising from real-world applications.

## Specific Expectations

### *Solving Quadratic Equations*

By the end of this course, students will:

- pose and solve problems involving quadratic relations arising from real-world applications and represented by tables of values and graphs (e.g., “From the graph of the height of a ball versus time, can you tell me how high the ball was thrown and the time when it hit the ground?”);
- represent situations (e.g., the area of a picture frame of variable width) using quadratic expressions in one variable, and expand and simplify quadratic expressions in one variable [e.g.,  $2x(x + 4) - (x + 3)^2$ ];\*
- factor quadratic expressions in one variable, including those for which  $a \neq 1$  (e.g.,  $3x^2 + 13x - 10$ ), differences of squares (e.g.,  $4x^2 - 25$ ), and perfect square trinomials (e.g.,  $9x^2 + 24x + 16$ ), by selecting and applying an appropriate strategy (**Sample problem:** Factor  $2x^2 - 12x + 10$ .);\*
- solve quadratic equations by selecting and applying a factoring strategy;
- determine, through investigation, and describe the connection between the factors used in solving a quadratic equation and the  $x$ -intercepts of the corresponding quadratic relation (**Sample problem:** The profit,  $P$ , of a video company, in thousands of dollars, is given by  $P = -5x^2 + 550x - 5000$ , where  $x$  is the amount spent on advertising, in thousands of dollars. Determine, by factoring and by graphing, the amount spent on advertising that will result in a profit of \$0. Describe the connection between the two strategies.);
- explore the algebraic development of the quadratic formula (e.g., given the algebraic development, connect the steps to a numerical example; follow a demonstration of the algebraic development, with technology, such as computer algebra systems, or without technology [student reproduction of the development of the general case is not required]), and apply the formula to solve quadratic equations, using technology;
- relate the real roots of a quadratic equation to the  $x$ -intercept(s) of the corresponding graph, and connect the number of real roots to the value of the discriminant (e.g., there are no real roots and no  $x$ -intercepts if  $b^2 - 4ac < 0$ );
- determine the real roots of a variety of quadratic equations (e.g.,  $100x^2 = 115x + 35$ ), and describe the advantages and disadvantages of each strategy (i.e., graphing; factoring; using the quadratic formula) (**Sample problem:** Generate 10 quadratic

\*The knowledge and skills described in this expectation may initially require the use of a variety of learning tools (e.g., computer algebra systems, algebra tiles, grid paper).

equations by randomly selecting integer values for  $a$ ,  $b$ , and  $c$  in  $ax^2 + bx + c = 0$ . Solve the equations using the quadratic formula. How many of the equations could you solve by factoring?).

### **Connecting Graphs and Equations of Quadratic Functions**

By the end of this course, students will:

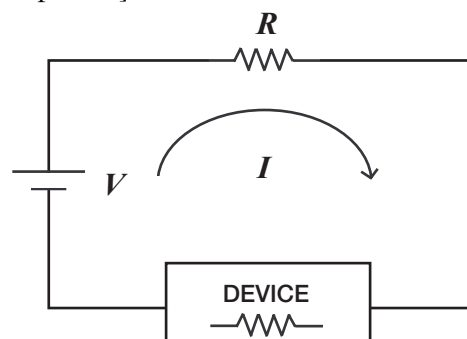
- explain the meaning of the term *function*, and distinguish a function from a relation that is not a function, through investigation of linear and quadratic relations using a variety of representations (i.e., tables of values, mapping diagrams, graphs, function machines, equations) and strategies (e.g., using the vertical line test) (**Sample problem:** Investigate, using numeric and graphical representations, whether the relation  $x = y^2$  is a function, and justify your reasoning.);
  - substitute into and evaluate linear and quadratic functions represented using function notation [e.g., evaluate  $f\left(\frac{1}{2}\right)$ , given  $f(x) = 2x^2 + 3x - 1$ ], including functions arising from real-world applications [**Sample problem:** The relationship between the selling price of a sleeping bag,  $s$  dollars, and the revenue at that selling price,  $r(s)$  dollars, is represented by the function  $r(s) = -10s^2 + 1500s$ . Evaluate, interpret, and compare  $r(29.95)$ ,  $r(60.00)$ ,  $r(75.00)$ ,  $r(90.00)$ , and  $r(130.00)$ .];
  - explain the meanings of the terms *domain* and *range*, through investigation using numeric, graphical, and algebraic representations of linear and quadratic functions, and describe the domain and range of a function appropriately (e.g., for  $y = x^2 + 1$ , the domain is the set of real numbers, and the range is  $y \geq 1$ );
  - explain any restrictions on the domain and the range of a quadratic function in contexts arising from real-world applications
- (**Sample problem:** A quadratic function represents the relationship between the height of a ball and the time elapsed since the ball was thrown. What physical factors will restrict the domain and range of the quadratic function?);
- determine, through investigation using technology, and describe the roles of  $a$ ,  $h$ , and  $k$  in quadratic functions of the form  $f(x) = a(x - h)^2 + k$  in terms of transformations on the graph of  $f(x) = x^2$  (i.e., translations; reflections in the  $x$ -axis; vertical stretches and compressions) [**Sample problem:** Investigate the graph  $f(x) = 3(x - h)^2 + 5$  for various values of  $h$ , using technology, and describe the effects of changing  $h$  in terms of a transformation.];
  - sketch graphs of  $g(x) = a(x - h)^2 + k$  by applying one or more transformations to the graph of  $f(x) = x^2$  [**Sample problem:** Transform the graph of  $f(x) = x^2$  to sketch the graphs of  $g(x) = x^2 - 4$  and  $h(x) = -2(x + 1)^2$ .];
  - express the equation of a quadratic function in the standard form  $f(x) = ax^2 + bx + c$ , given the vertex form  $f(x) = a(x - h)^2 + k$ , and verify, using graphing technology, that these forms are equivalent representations [**Sample problem:** Given the vertex form  $f(x) = 3(x - 1)^2 + 4$ , express the equation in standard form. Use technology to compare the graphs of these two forms of the equation.];
  - express the equation of a quadratic function in the vertex form  $f(x) = a(x - h)^2 + k$ , given the standard form  $f(x) = ax^2 + bx + c$ , by completing the square (e.g., using algebra tiles or diagrams; algebraically), including cases where  $\frac{b}{a}$  is a simple rational number (e.g.,  $\frac{1}{2}$ , 0.75), and verify, using graphing technology, that these forms are equivalent representations;

- sketch graphs of quadratic functions in the factored form  $f(x) = a(x - r)(x - s)$  by using the  $x$ -intercepts to determine the vertex;
- describe the information (e.g., maximum, intercepts) that can be obtained by inspecting the standard form  $f(x) = ax^2 + bx + c$ , the vertex form  $f(x) = a(x - h)^2 + k$ , and the factored form  $f(x) = a(x - r)(x - s)$  of a quadratic function;
- sketch the graph of a quadratic function whose equation is given in the standard form  $f(x) = ax^2 + bx + c$  by using a suitable strategy (e.g., completing the square and finding the vertex; factoring, if possible, to locate the  $x$ -intercepts), and identify the key features of the graph (e.g., the vertex, the  $x$ - and  $y$ -intercepts, the equation of the axis of symmetry, the intervals where the function is positive or negative, the intervals where the function is increasing or decreasing).
- determine, through investigation using a variety of strategies (e.g., applying properties of quadratic functions such as the  $x$ -intercepts and the vertex; using transformations), the equation of the quadratic function that best models a suitable data set graphed on a scatter plot, and compare this equation to the equation of a curve of best fit generated with technology (e.g., graphing software, graphing calculator);
- solve problems arising from real-world applications, given the algebraic representation of a quadratic function (e.g., given the equation of a quadratic function representing the height of a ball over elapsed time, answer questions that involve the maximum height of the ball, the length of time needed for the ball to touch the ground, and the time interval when the ball is higher than a given measurement) [**Sample problem:** In a DC electrical circuit, the relationship between the power used by a device,  $P$  (in watts, W), the electric potential difference (voltage),  $V$  (in volts, V), the current,  $I$  (in amperes, A), and the resistance,  $R$  (in ohms,  $\Omega$ ), is represented by the formula  $P = IV - I^2R$ . Represent graphically and algebraically the relationship between the power and the current when the electric potential difference is 24 V and the resistance is 1.5  $\Omega$ . Determine the current needed in order for the device to use the maximum amount of power.].

### Solving Problems Involving Quadratic Functions

By the end of this course, students will:

- collect data that can be modelled as a quadratic function, through investigation with and without technology, from primary sources, using a variety of tools (e.g., concrete materials; measurement tools such as measuring tapes, electronic probes, motion sensors), or from secondary sources (e.g., websites such as Statistics Canada, E-STAT), and graph the data (**Sample problem:** When a  $3 \times 3 \times 3$  cube made up of  $1 \times 1 \times 1$  cubes is dipped into red paint, 6 of the smaller cubes will have 1 face painted. Investigate the number of smaller cubes with 1 face painted as a function of the edge length of the larger cube, and graph the function.);



# Exponential Functions

## Overall Expectations

By the end of this course, students will:

- simplify and evaluate numerical expressions involving exponents, and make connections between the numeric, graphical, and algebraic representations of exponential functions;
- identify and represent exponential functions, and solve problems involving exponential functions, including those arising from real-world applications;
- demonstrate an understanding of compound interest and annuities, and solve related problems.

## Specific Expectations

### *Connecting Graphs and Equations of Exponential Functions*

By the end of this course, students will:

- determine, through investigation using a variety of tools (e.g., calculator, paper and pencil, graphing technology) and strategies (e.g., patterning; finding values from a graph; interpreting the exponent laws), the value of a power with a rational exponent (i.e.,  $x^{\frac{m}{n}}$ , where  $x > 0$  and  $m$  and  $n$  are integers) (**Sample problem:** The exponent laws suggest that  $4^{\frac{1}{2}} \times 4^{\frac{1}{2}} = 4^1$ . What value would you assign to  $4^{\frac{1}{2}}$ ? What value would you assign to  $27^{\frac{1}{3}}$ ? Explain your reasoning. Extend your reasoning to make a generalization about the meaning of  $x^{\frac{1}{n}}$ , where  $x > 0$  and  $n$  is a natural number.);
- evaluate, with and without technology, numerical expressions containing integer and rational exponents and rational bases [e.g.,  $2^{-3}$ ,  $(-6)^3$ ,  $4^{\frac{1}{2}}$ ,  $1.01^{120}$ ];
- graph, with and without technology, an exponential relation, given its equation in the form  $y = a^x$  ( $a > 0$ ,  $a \neq 1$ ), define this relation as the function  $f(x) = a^x$ , and explain why it is a function;
- determine, through investigation, and describe key properties relating to domain and range, intercepts, increasing/decreasing intervals, and asymptotes (e.g., the domain is the set of real numbers; the range is the set of positive real numbers; the function either increases or decreases throughout its domain) for exponential functions represented in a variety of ways [e.g., tables of values, mapping diagrams, graphs, equations of the form  $f(x) = a^x$  ( $a > 0$ ,  $a \neq 1$ ), function machines] [**Sample problem:** Graph  $f(x) = 2^x$ ,  $g(x) = 3^x$ , and  $h(x) = 0.5^x$  on the same set of axes. Make comparisons between the graphs, and explain the relationship between the  $y$ -intercepts.];
- determine, through investigation (e.g., by patterning with and without a calculator), the exponent rules for multiplying and dividing numerical expressions involving exponents [e.g.,  $(\frac{1}{2})^3 \times (\frac{1}{2})^2$ ], and the exponent rule for simplifying numerical expressions involving a power of a power [e.g.,  $(5^3)^2$ ], and use the rules to simplify numerical expressions containing integer exponents [e.g.,  $(2^3)(2^5) = 2^8$ ];
- distinguish exponential functions from linear and quadratic functions by making comparisons in a variety of ways (e.g., comparing rates of change using finite differences in tables of values; identifying a constant ratio in a table of values; inspecting graphs; comparing equations), within the same context when possible (e.g., simple interest and compound interest; population growth) [**Sample problem:** Explain in a variety of ways how you can distinguish



the exponential function  $f(x) = 2^x$  from the quadratic function  $f(x) = x^2$  and the linear function  $f(x) = 2x$ ].

### **Solving Problems Involving Exponential Functions**

By the end of this course, students will:

- collect data that can be modelled as an exponential function, through investigation with and without technology, from primary sources, using a variety of tools (e.g., concrete materials such as number cubes, coins; measurement tools such as electronic probes), or from secondary sources (e.g., websites such as Statistics Canada, E-STAT), and graph the data (**Sample problem:** Collect data and graph the cooling curve representing the relationship between temperature and time for hot water cooling in a porcelain mug. Predict the shape of the cooling curve when hot water cools in an insulated mug. Test your prediction.);
- identify exponential functions, including those that arise from real-world applications involving growth and decay (e.g., radioactive decay, population growth, cooling rates, pressure in a leaking tire), given various representations (i.e., tables of values, graphs, equations), and explain any restrictions that the context places on the domain and range (e.g., ambient temperature limits the range for a cooling curve);
- solve problems using given graphs or equations of exponential functions arising from a variety of real-world applications (e.g., radioactive decay, population growth, height of a bouncing ball, compound interest) by interpreting the graphs or by substituting values for the exponent into the equations [**Sample problem:** The temperature of a cooling liquid over time can be modelled by the exponential function  $T(x) = 60 \left(\frac{1}{2}\right)^{\frac{x}{30}} + 20$ ,

where  $T(x)$  is the temperature, in degrees Celsius, and  $x$  is the elapsed time, in minutes. Graph the function and determine how long it takes for the temperature to reach 28°C.].

### **Solving Financial Problems Involving Exponential Functions**

By the end of this course, students will:

- compare, using a table of values and graphs, the simple and compound interest earned for a given principal (i.e., investment) and a fixed interest rate over time (**Sample problem:** Compare, using tables of values and graphs, the amounts after each of the first five years for a \$1000 investment at 5% simple interest per annum and a \$1000 investment at 5% interest per annum, compounded annually.);
- solve problems, using a scientific calculator, that involve the calculation of the amount,  $A$  (also referred to as future value,  $FV$ ), and the principal,  $P$  (also referred to as present value,  $PV$ ), using the compound interest formula in the form  $A = P(1 + i)^n$  [or  $FV = PV(1 + i)^n$ ] (**Sample problem:** Calculate the amount if \$1000 is invested for three years at 6% per annum, compounded quarterly.);
- determine, through investigation (e.g., using spreadsheets and graphs), that compound interest is an example of exponential growth [e.g., the formulas for compound interest,  $A = P(1 + i)^n$ , and present value,  $PV = A(1 + i)^{-n}$ , are exponential functions, where the number of compounding periods,  $n$ , varies] [**Sample problem:** Describe an investment that could be represented by the function  $f(x) = 500(1.01)^x$ .];
- solve problems, using a TVM Solver in a graphing calculator or on a website, that involve the calculation of the interest rate per compounding period,  $i$ , or the number

- of compounding periods,  $n$ , in the compound interest formula  $A = P(1 + i)^n$  [or  $FV = PV(1 + i)^n$  ], (**Sample problem:** Use the TVM Solver in a graphing calculator to determine the time it takes to double an investment in an account that pays interest of 4% per annum, compounded semi-annually.);
- explain the meaning of the term *annuity*, through investigation of numerical and graphical representations using technology;
  - determine, through investigation using technology (e.g., the TVM Solver in a graphing calculator; online tools), the effects of changing the conditions (i.e., the payments, the frequency of the payments, the interest rate, the compounding period) of ordinary annuities in situations where the compounding period and the payment period are the same (e.g., long-term savings plans, loans) (**Sample problem:** Compare the amounts at age 65 that would result from making an annual deposit of \$1000 starting at age 20, or from making an annual deposit of \$3000 starting at age 50, to an RRSP that earns 6% interest per annum, compounded annually. What is the total of the deposits in each situation?);
  - solve problems, using technology (e.g., scientific calculator, spreadsheet, graphing calculator), that involve the amount, the present value, and the regular payment of an ordinary annuity in situations where the compounding period and the payment period are the same (e.g., calculate the total interest paid over the life of a loan, using a spreadsheet, and compare the total interest with the original principal of the loan).

## Trigonometric Functions

### Overall Expectations

By the end of this course, students will:

- solve problems involving trigonometry in acute triangles using the sine law and the cosine law, including problems arising from real-world applications;
- demonstrate an understanding of periodic relationships and the sine function, and make connections between the numeric, graphical, and algebraic representations of sine functions;
- identify and represent sine functions, and solve problems involving sine functions, including those arising from real-world applications.

### Specific Expectations

#### *Applying the Sine Law and the Cosine Law in Acute Triangles*

By the end of this course, students will:

- solve problems, including those that arise from real-world applications (e.g., surveying, navigation), by determining the measures of the sides and angles of right triangles using the primary trigonometric ratios;
- solve problems involving two right triangles in two dimensions (**Sample problem:** A helicopter hovers 500 m above a long straight road. Ahead of the helicopter on the road are two trucks. The angles of depression of the two trucks from the helicopter are  $60^\circ$  and  $20^\circ$ . How far apart are the two trucks?);
- verify, through investigation using technology (e.g., dynamic geometry software, spreadsheet), the sine law and the cosine law (e.g., compare, using dynamic geometry software, the ratios  $\frac{a}{\sin A}$ ,  $\frac{b}{\sin B}$ , and  $\frac{c}{\sin C}$  in triangle ABC while dragging one of the vertices);
- describe conditions that guide when it is appropriate to use the sine law or the cosine law, and use these laws to calculate sides and angles in acute triangles;
- solve problems that require the use of the sine law or the cosine law in acute triangles, including problems arising from

real-world applications (e.g., surveying; navigation; building construction).

#### *Connecting Graphs and Equations of Sine Functions*

By the end of this course, students will:

- describe key properties (e.g., cycle, amplitude, period) of periodic functions arising from real-world applications (e.g., natural gas consumption in Ontario, tides in the Bay of Fundy), given a numerical or graphical representation;
- predict, by extrapolating, the future behaviour of a relationship modelled using a numeric or graphical representation of a periodic function (e.g., predicting hours of daylight on a particular date from previous measurements; predicting natural-gas consumption in Ontario from previous consumption);
- make connections between the sine ratio and the sine function by graphing the relationship between angles from  $0^\circ$  to  $360^\circ$  and the corresponding sine ratios, with or without technology (e.g., by generating a table of values using a calculator; by unwrapping the unit circle), defining this relationship as the function  $f(x) = \sin x$ , and explaining why it is a function;
- sketch the graph of  $f(x) = \sin x$  for angle measures expressed in degrees, and determine and describe its key properties (i.e., cycle, domain, range, intercepts, amplitude,

- period, maximum and minimum values, increasing/decreasing intervals);
- make connections, through investigation with technology, between changes in a real-world situation that can be modelled using a periodic function and transformations of the corresponding graph (e.g., investigating the connection between variables for a swimmer swimming lengths of a pool and transformations of the graph of distance from the starting point versus time) (**Sample problem:** Generate a sine curve by walking a circle of two-metre diameter in front of a motion sensor. Describe how the following changes in the motion change the graph: starting at a different point on the circle; starting a greater distance from the motion sensor; changing direction; increasing the radius of the circle; and increasing the speed);
  - determine, through investigation using technology, and describe the roles of the parameters  $a$ ,  $c$ , and  $d$  in functions in the form  $f(x) = a \sin x$ ,  $f(x) = \sin x + c$ , and  $f(x) = \sin(x - d)$  in terms of transformations on the graph of  $f(x) = \sin x$  with angles expressed in degrees (i.e., translations; reflections in the  $x$ -axis; vertical stretches and compressions);
  - sketch graphs of  $f(x) = a \sin x$ ,  $f(x) = \sin x + c$ , and  $f(x) = \sin(x - d)$  by applying transformations to the graph of  $f(x) = \sin x$ , and state the domain and range of the transformed functions [**Sample problem:** Transform the graph of  $f(x) = \sin x$  to sketch the graphs of  $g(x) = -2\sin x$  and  $h(x) = \sin(x - 180^\circ)$ , and state the domain and range of each function.].
- Solving Problems Involving Sine Functions**  
By the end of this course, students will:
- collect data that can be modelled as a sine function (e.g., voltage in an AC circuit, sound waves), through investigation with and without technology, from primary sources, using a variety of tools (e.g., concrete materials; measurement tools such as motion sensors), or from secondary sources (e.g., websites such as Statistics Canada, E-STAT), and graph the data (**Sample problem:** Measure and record distance-time data for a swinging pendulum, using a motion sensor or other measurement tools, and graph the data.);
  - identify sine functions, including those that arise from real-world applications involving periodic phenomena, given various representations (i.e., tables of values, graphs, equations), and explain any restrictions that the context places on the domain and range;
  - pose and solve problems based on applications involving a sine function by using a given graph or a graph generated with technology from its equation [**Sample problem:** The height above the ground of a rider on a Ferris wheel can be modelled by the sine function  $h(x) = 25 \sin(x - 90^\circ) + 27$ , where  $h(x)$  is the height, in metres, and  $x$  is the angle, in degrees, that the radius to the rider makes with the horizontal. Graph the function, using graphing technology in degree mode, and determine the maximum and minimum heights of the rider, and the measures of the angle when the height of the rider is 40 m.].

# Foundations for College Mathematics, Grade 11, College Preparation

(MBF3C)

This course enables students to broaden their understanding of mathematics as a problem-solving tool in the real world. Students will extend their understanding of quadratic relations; investigate situations involving exponential growth; solve problems involving compound interest; solve financial problems connected with vehicle ownership; develop their ability to reason by collecting, analysing, and evaluating data involving one variable; connect probability and statistics; and solve problems in geometry and trigonometry. Students will consolidate their mathematical skills as they solve problems and communicate their thinking.

**Prerequisite:** Foundations of Mathematics, Grade 10, Applied

**Mathematical process expectations.** The mathematical processes are to be integrated into student learning in all areas of this course.

**Throughout this course, students will:**

## PROBLEM SOLVING

- develop, select, apply, compare, and adapt a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;

## REASONING AND PROVING

- develop and apply reasoning skills (e.g., use of inductive reasoning, deductive reasoning, and counter-examples; construction of proofs) to make mathematical conjectures, assess conjectures, and justify conclusions, and plan and construct organized mathematical arguments;

## REFLECTING

- demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by assessing the effectiveness of strategies and processes used, by proposing alternative approaches, by judging the reasonableness of results, by verifying solutions);

## SELECTING TOOLS AND COMPUTATIONAL STRATEGIES

- select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;

## CONNECTING

- make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, current events, art and culture, sports);

## REPRESENTING

- create a variety of representations of mathematical ideas (e.g., numeric, geometric, algebraic, graphical, pictorial representations; onscreen dynamic representations), connect and compare them, and select and apply the appropriate representations to solve problems;

## COMMUNICATING

- communicate mathematical thinking orally, visually, and in writing, using precise mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.

## Mathematical Models

### Overall Expectations

By the end of this course, students will:

- make connections between the numeric, graphical, and algebraic representations of quadratic relations, and use the connections to solve problems;
- demonstrate an understanding of exponents, and make connections between the numeric, graphical, and algebraic representations of exponential relations;
- describe and represent exponential relations, and solve problems involving exponential relations arising from real-world applications.

### Specific Expectations

#### *Connecting Graphs and Equations of Quadratic Relations*

By the end of this course, students will:

- construct tables of values and graph quadratic relations arising from real-world applications (e.g., dropping a ball from a given height; varying the edge length of a cube and observing the effect on the surface area of the cube);
- determine and interpret meaningful values of the variables, given a graph of a quadratic relation arising from a real-world application (**Sample problem:** Under certain conditions, there is a quadratic relation between the profit of a manufacturing company and the number of items it produces. Explain how you could interpret a graph of the relation to determine the numbers of items produced for which the company makes a profit and to determine the maximum profit the company can make.);
- determine, through investigation using technology, and describe the roles of  $a$ ,  $h$ , and  $k$  in quadratic relations of the form  $y = a(x - h)^2 + k$  in terms of transformations on the graph of  $y = x^2$  (i.e., translations; reflections in the  $x$ -axis; vertical stretches and compressions) [**Sample problem:** Investigate the graph  $y = 3(x - h)^2 + 5$  for various values of  $h$ , using technology, and describe the effects of changing  $h$  in terms of a transformation.];
- sketch graphs of quadratic relations represented by the equation  $y = a(x - h)^2 + k$  (e.g., using the vertex and at least one point on each side of the vertex; applying one or more transformations to the graph of  $y = x^2$ );
- expand and simplify quadratic expressions in one variable involving multiplying binomials [e.g.,  $(\frac{1}{2}x + 1)(3x - 2)$ ] or squaring a binomial [e.g.,  $5(3x - 1)^2$ ], using a variety of tools (e.g., paper and pencil, algebra tiles, computer algebra systems);
- express the equation of a quadratic relation in the standard form  $y = ax^2 + bx + c$ , given the vertex form  $y = a(x - h)^2 + k$ , and verify, using graphing technology, that these forms are equivalent representations [**Sample problem:** Given the vertex form  $y = 3(x - 1)^2 + 4$ , express the equation in standard form. Use technology to compare the graphs of these two forms of the equation.];
- factor trinomials of the form  $ax^2 + bx + c$ , where  $a = 1$  or where  $a$  is the common factor, by various methods;
- determine, through investigation, and describe the connection between the factors of a quadratic expression and the  $x$ -intercepts of the graph of the corresponding quadratic relation (**Sample problem:** Investigate

the relationship between the factored form of  $3x^2 + 15x + 12$  and the  $x$ -intercepts of  $y = 3x^2 + 15x + 12$ .);

- solve problems, using an appropriate strategy (i.e., factoring, graphing), given equations of quadratic relations, including those that arise from real-world applications (e.g., break-even point) (**Sample problem:** On planet X, the height,  $h$  metres, of an object fired upward from the ground at 48 m/s is described by the equation  $h = 48t - 16t^2$ , where  $t$  seconds is the time since the object was fired upward. Determine the maximum height of the object, the times at which the object is 32 m above the ground, and the time at which the object hits the ground.).)

### **Connecting Graphs and Equations of Exponential Relations**

By the end of this course, students will:

- determine, through investigation using a variety of tools and strategies (e.g., graphing with technology; looking for patterns in tables of values), and describe the meaning of negative exponents and of zero as an exponent;
- evaluate, with and without technology, numerical expressions containing integer exponents and rational bases (e.g.,  $2^{-3}$ ,  $6^3$ ,  $3456^0$ ,  $1.03^{10}$ );
- determine, through investigation (e.g., by patterning with and without a calculator), the exponent rules for multiplying and dividing numerical expressions involving exponents [e.g.,  $(\frac{1}{2})^3 \times (\frac{1}{2})^2$ ], and the exponent rule for simplifying numerical expressions involving a power of a power [e.g.,  $(5^3)^2$ ];
- graph simple exponential relations, using paper and pencil, given their equations [e.g.,  $y = 2^x$ ,  $y = 10^x$ ,  $y = (\frac{1}{2})^x$ ];

- make and describe connections between representations of an exponential relation (i.e., numeric in a table of values; graphical; algebraic);
- distinguish exponential relations from linear and quadratic relations by making comparisons in a variety of ways (e.g., comparing rates of change using finite differences in tables of values; inspecting graphs; comparing equations), within the same context when possible (e.g., simple interest and compound interest; population growth) (**Sample problem:** Explain in a variety of ways how you can distinguish exponential growth represented by  $y = 2^x$  from quadratic growth represented by  $y = x^2$  and linear growth represented by  $y = 2x$ .).)

### **Solving Problems Involving Exponential Relations**

By the end of this course, students will:

- collect data that can be modelled as an exponential relation, through investigation with and without technology, from primary sources, using a variety of tools (e.g., concrete materials such as number cubes, coins; measurement tools such as electronic probes), or from secondary sources (e.g., websites such as Statistics Canada, E-STAT), and graph the data (**Sample problem:** Collect data and graph the cooling curve representing the relationship between temperature and time for hot water cooling in a porcelain mug. Predict the shape of the cooling curve when hot water cools in an insulated mug. Test your prediction.);
- describe some characteristics of exponential relations arising from real-world applications (e.g., bacterial growth, drug absorption) by using tables of values (e.g., to show a constant ratio, or multiplicative growth or decay) and graphs (e.g., to show, with technology, that there is no maximum or minimum value);

- pose and solve problems involving exponential relations arising from a variety of real-world applications (e.g., population growth, radioactive decay, compound interest) by using a given graph or a graph generated with technology from a given equation (**Sample problem:** Given a graph of the population of a bacterial colony versus time, determine the change in population in the first hour.);
- solve problems using given equations of exponential relations arising from a variety of real-world applications (e.g., radioactive decay, population growth, height of a bouncing ball, compound interest) by substituting values for the exponent into the equations (**Sample problem:** The height,  $h$  metres, of a ball after  $n$  bounces is given by the equation  $h = 2(0.6)^n$ . Determine the height of the ball after 3 bounces.).



## Personal Finance

### Overall Expectations

By the end of this course, students will:

- compare simple and compound interest, relate compound interest to exponential growth, and solve problems involving compound interest;
- compare services available from financial institutions, and solve problems involving the cost of making purchases on credit;
- interpret information about owning and operating a vehicle, and solve problems involving the associated costs.

### Specific Expectations

#### *Solving Problems Involving Compound Interest*

By the end of this course, students will:

- determine, through investigation using technology, the compound interest for a given investment, using repeated calculations of simple interest, and compare, using a table of values and graphs, the simple and compound interest earned for a given principal (i.e., investment) and a fixed interest rate over time (**Sample problem:** Compare, using tables of values and graphs, the amounts after each of the first five years for a \$1000 investment at 5% simple interest per annum and a \$1000 investment at 5% interest per annum, compounded annually.);
- determine, through investigation (e.g., using spreadsheets and graphs), and describe the relationship between compound interest and exponential growth;
- solve problems, using a scientific calculator, that involve the calculation of the amount,  $A$  (also referred to as future value,  $FV$ ), and the principal,  $P$  (also referred to as present value,  $PV$ ), using the compound interest formula in the form  $A = P(1 + i)^n$  [or  $FV = PV(1 + i)^n$ ] (**Sample problem:** Calculate the amount if \$1000 is invested for 3 years at 6% per annum, compounded quarterly.);
- calculate the total interest earned on an investment or paid on a loan by determining the difference between the amount and the principal [e.g., using  $I = A - P$  (or  $I = FV - PV$ )];
- solve problems, using a TVM Solver in a graphing calculator or on a website, that involve the calculation of the interest rate per compounding period,  $i$ , or the number of compounding periods,  $n$ , in the compound interest formula  $A = P(1 + i)^n$  [or  $FV = PV(1 + i)^n$ ] (**Sample problem:** Use the TVM Solver in a graphing calculator to determine the time it takes to double an investment in an account that pays interest of 4% per annum, compounded semi-annually.);
- determine, through investigation using technology (e.g., a TVM Solver in a graphing calculator or on a website), the effect on the future value of a compound interest investment or loan of changing the total length of time, the interest rate, or the compounding period (**Sample problem:** Investigate whether doubling the interest rate will halve the time it takes for an investment to double.).

### **Comparing Financial Services**

By the end of this course, students will:

- gather, interpret, and compare information about the various savings alternatives commonly available from financial institutions (e.g., savings and chequing accounts, term investments), the related costs (e.g., cost of cheques, monthly statement fees, early withdrawal penalties), and possible ways of reducing the costs (e.g., maintaining a minimum balance in a savings account; paying a monthly flat fee for a package of services);
- gather and interpret information about investment alternatives (e.g., stocks, mutual funds, real estate, GICs, savings accounts), and compare the alternatives by considering the risk and the rate of return;
- gather, interpret, and compare information about the costs (e.g., user fees, annual fees, service charges, interest charges on overdue balances) and incentives (e.g., loyalty rewards; philanthropic incentives, such as support for Olympic athletes or a Red Cross disaster relief fund) associated with various credit cards and debit cards;
- gather, interpret, and compare information about current credit card interest rates and regulations, and determine, through investigation using technology, the effects of delayed payments on a credit card balance;
- solve problems involving applications of the compound interest formula to determine the cost of making a purchase on credit (**Sample problem:** Using information gathered about the interest rates and regulations for two different credit cards, compare the costs of purchasing a \$1500 computer with each card if the full amount is paid 55 days later.).

### **Owning and Operating a Vehicle**

By the end of this course, students will:

- gather and interpret information about the procedures and costs involved in insuring a vehicle (e.g., car, motorcycle, snowmobile) and the factors affecting insurance rates (e.g., gender, age, driving record, model of vehicle, use of vehicle), and compare the insurance costs for different categories of drivers and for different vehicles (**Sample problem:** Use automobile insurance websites to investigate the degree to which the type of car and the age and gender of the driver affect insurance rates.);
- gather, interpret, and compare information about the procedures and costs (e.g., monthly payments, insurance, depreciation, maintenance, miscellaneous expenses) involved in buying or leasing a new vehicle or buying a used vehicle (**Sample problem:** Compare the costs of buying a new car, leasing the same car, and buying an older model of the same car.);
- solve problems, using technology (e.g., calculator, spreadsheet), that involve the fixed costs (e.g., licence fee, insurance) and variable costs (e.g., maintenance, fuel) of owning and operating a vehicle (**Sample problem:** The rate at which a car consumes gasoline depends on the speed of the car. Use a given graph of gasoline consumption, in litres per 100 km, versus speed, in kilometres per hour, to determine how much gasoline is used to drive 500 km at speeds of 80 km/h, 100 km/h, and 120 km/h. Use the current price of gasoline to calculate the cost of driving 500 km at each of these speeds.).

# Geometry and Trigonometry

## Overall Expectations

By the end of this course, students will:

- represent, in a variety of ways, two-dimensional shapes and three-dimensional figures arising from real-world applications, and solve design problems;
- solve problems involving trigonometry in acute triangles using the sine law and the cosine law, including problems arising from real-world applications.

## Specific Expectations

### *Representing Two-Dimensional Shapes and Three-Dimensional Figures*

By the end of this course, students will:

- identify real-world applications of geometric shapes and figures, through investigation (e.g., by importing digital photos into dynamic geometry software), in a variety of contexts (e.g., product design, architecture, fashion), and explain these applications (e.g., one reason that sewer covers are round is to prevent them from falling into the sewer during removal and replacement) (**Sample problem:** Explain why rectangular prisms are used for packaging many products.);
- represent three-dimensional objects, using concrete materials and design or drawing software, in a variety of ways (e.g., orthographic projections [i.e., front, side, and top views]; perspective isometric drawings; scale models);
- create nets, plans, and patterns from physical models arising from a variety of real-world applications (e.g., fashion design; interior decorating; building construction), by applying the metric and imperial systems and using design or drawing software;
- solve design problems that satisfy given constraints (e.g., design a rectangular berm that would contain all the oil that could leak from a cylindrical storage tank of a given height and radius), using physical models (e.g., built from popsicle sticks,

cardboard, duct tape) or drawings (e.g., made using design or drawing software), and state any assumptions made (**Sample problem:** Design and construct a model boat that can carry the most pennies, using one sheet of 8.5 in × 11 in card stock, no more than five popsicle sticks, and some adhesive tape or glue.).

### *Applying the Sine Law and the Cosine Law in Acute Triangles*

By the end of this course, students will:

- solve problems, including those that arise from real-world applications (e.g., surveying, navigation), by determining the measures of the sides and angles of right triangles using the primary trigonometric ratios;
- verify, through investigation using technology (e.g., dynamic geometry software, spreadsheet), the sine law and the cosine law (e.g., compare, using dynamic geometry software, the ratios  $\frac{a}{\sin A}$ ,  $\frac{b}{\sin B}$ , and  $\frac{c}{\sin C}$  in triangle ABC while dragging one of the vertices);
- describe conditions that guide when it is appropriate to use the sine law or the cosine law, and use these laws to calculate sides and angles in acute triangles;
- solve problems that arise from real-world applications involving metric and imperial measurements and that require the use of the sine law or the cosine law in acute triangles.

## Data Management

### Overall Expectations

By the end of this course, students will:

- solve problems involving one-variable data by collecting, organizing, analysing, and evaluating data;
- determine and represent probability, and identify and interpret its applications.

### Specific Expectations

#### *Working With One-Variable Data*

By the end of this course, students will:

- identify situations involving one-variable data (i.e., data about the frequency of a given occurrence), and design questionnaires (e.g., for a store to determine which CDs to stock; for a radio station to choose which music to play) or experiments (e.g., counting, taking measurements) for gathering one-variable data, giving consideration to ethics, privacy, the need for honest responses, and possible sources of bias (**Sample problem:** One lane of a three-lane highway is being restricted to vehicles with at least two passengers to reduce traffic congestion. Design an experiment to collect one-variable data to decide whether traffic congestion is actually reduced.);
- collect one-variable data from secondary sources (e.g., Internet databases), and organize and store the data using a variety of tools (e.g., spreadsheets, dynamic statistical software);
- explain the distinction between the terms *population* and *sample*, describe the characteristics of a good sample, and explain why sampling is necessary (e.g., time, cost, or physical constraints) (**Sample problem:** Explain the terms *sample* and *population* by giving examples within your school and your community.);
- describe and compare sampling techniques (e.g., random, stratified, clustered, convenience, voluntary); collect one-variable data from primary sources, using appropriate sampling techniques in a variety of real-world situations; and organize and store the data;
- identify different types of one-variable data (i.e., categorical, discrete, continuous), and represent the data, with and without technology, in appropriate graphical forms (e.g., histograms, bar graphs, circle graphs, pictographs);
- identify and describe properties associated with common distributions of data (e.g., normal, bimodal, skewed);
- calculate, using formulas and/or technology (e.g., dynamic statistical software, spreadsheet, graphing calculator), and interpret measures of central tendency (i.e., mean, median, mode) and measures of spread (i.e., range, standard deviation);
- explain the appropriate use of measures of central tendency (i.e., mean, median, mode) and measures of spread (i.e., range, standard deviation) (**Sample problem:** Explain whether the mean or the median of your course marks would be the more appropriate representation of your achievement. Describe the additional information that the standard deviation of your course marks would provide.);

- compare two or more sets of one-variable data, using measures of central tendency and measures of spread (**Sample problem:** Use measures of central tendency and measures of spread to compare data that show the lifetime of an economy light bulb with data that show the lifetime of a long-life light bulb.);
- solve problems by interpreting and analysing one-variable data collected from secondary sources.

### **Applying Probability**

By the end of this course, students will:

- identify examples of the use of probability in the media and various ways in which probability is represented (e.g., as a fraction, as a percent, as a decimal in the range 0 to 1);
- determine the theoretical probability of an event (i.e., the ratio of the number of favourable outcomes to the total number of possible outcomes, where all outcomes are equally likely), and represent the probability in a variety of ways (e.g., as a fraction, as a percent, as a decimal in the range 0 to 1);
- perform a probability experiment (e.g., tossing a coin several times), represent the results using a frequency distribution, and use the distribution to determine the experimental probability of an event;
- compare, through investigation, the theoretical probability of an event with the experimental probability, and explain why they might differ (**Sample problem:** If you toss 10 coins repeatedly, explain why 5 heads are unlikely to result from every toss.);
- determine, through investigation using class-generated data and technology-based simulation models (e.g., using a random-number generator on a spreadsheet or on a graphing calculator), the tendency of experimental probability to approach theoretical probability as the number of trials in an experiment increases (e.g., “If I simulate tossing a coin 1000 times using technology, the experimental probability that I calculate for tossing tails is likely to be closer to the theoretical probability than if I only simulate tossing the coin 10 times”) (**Sample problem:** Calculate the theoretical probability of rolling a 2 on a number cube. Simulate rolling a number cube, and use the simulation to calculate the experimental probability of rolling a 2 after 10, 20, 30, . . . , 200 trials. Graph the experimental probability versus the number of trials, and describe any trend.);
- interpret information involving the use of probability and statistics in the media, and make connections between probability and statistics (e.g., statistics can be used to generate probabilities).

# Mathematics for Work and Everyday Life, Grade 11, Workplace Preparation

(MEL3E)

This course enables students to broaden their understanding of mathematics as it is applied in the workplace and daily life. Students will solve problems associated with earning money, paying taxes, and making purchases; apply calculations of simple and compound interest in saving, investing, and borrowing; and calculate the costs of transportation and travel in a variety of situations. Students will consolidate their mathematical skills as they solve problems and communicate their thinking.

**Prerequisite:** Principles of Mathematics, Grade 9, Academic, or Foundations of Mathematics, Grade 9, Applied, or a ministry-approved locally developed Grade 10 mathematics course

**Mathematical process expectations.** The mathematical processes are to be integrated into student learning in all areas of this course.

### Throughout this course, students will:

#### PROBLEM SOLVING

- develop, select, apply, compare, and adapt a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;

#### REASONING AND PROVING

- develop and apply reasoning skills (e.g., use of inductive reasoning, deductive reasoning, and counter-examples; construction of proofs) to make mathematical conjectures, assess conjectures, and justify conclusions, and plan and construct organized mathematical arguments;

#### REFLECTING

- demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by assessing the effectiveness of strategies and processes used, by proposing alternative approaches, by judging the reasonableness of results, by verifying solutions);

#### SELECTING TOOLS AND COMPUTATIONAL STRATEGIES

- select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;

#### CONNECTING

- make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, current events, art and culture, sports);

#### REPRESENTING

- create a variety of representations of mathematical ideas (e.g., numeric, geometric, algebraic, graphical, pictorial representations; onscreen dynamic representations), connect and compare them, and select and apply the appropriate representations to solve problems;

#### COMMUNICATING

- communicate mathematical thinking orally, visually, and in writing, using precise mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.

## Earning and Purchasing

### Overall Expectations

By the end of this course, students will:

- interpret information about different types of remuneration, and solve problems and make decisions involving different remuneration methods;
- demonstrate an understanding of payroll deductions and their impact on purchasing power;
- demonstrate an understanding of the factors and methods involved in making and justifying informed purchasing decisions.

### Specific Expectations

#### *Earning*

By the end of this course, students will:

- gather, interpret, and compare information about the components of total earnings (e.g., salary, benefits, vacation pay, profit-sharing) in different occupations;
- gather, interpret, and describe information about different remuneration methods (e.g., hourly rate, overtime rate, job or project rate, commission, salary, gratuities) and remuneration schedules (e.g. weekly, biweekly, semi-monthly, monthly);
- describe the effects of different remuneration methods and schedules on decisions related to personal spending habits (e.g., the timing of a major purchase; the scheduling of mortgage payments and other bill payments);
- solve problems, using technology (e.g., calculator, spreadsheet), and make decisions involving different remuneration methods and schedules (**Sample problem:** Two sales positions are available in sportswear stores. One pays an hourly rate of \$11.25 for 40 h per week. The other pays a weekly salary of \$375 for the same number of hours, plus a commission of 5% of sales. Under what conditions would each position be the better choice?).

#### *Describing Purchasing Power*

By the end of this course, students will:

- gather, interpret, and describe information about government payroll deductions (i.e., CPP, EI, income tax) and other payroll deductions (e.g., contributions to pension plans other than CPP; union dues; charitable donations; benefit-plan contributions);
- estimate and compare, using current secondary data (e.g., federal tax tables), the percent of total earnings deducted through government payroll deductions for various benchmarks (e.g., \$15 000, \$20 000, \$25 000) (**Sample problem:** Compare the percent of total earnings deducted through government payroll deductions for total earnings of \$15 000 and \$45 000.);
- describe the relationship between gross pay, net pay, and payroll deductions (i.e., net pay is gross pay less government payroll deductions and any other payroll deductions), and estimate net pay in various situations;
- describe and compare the purchasing power and living standards associated with relevant occupations of interest.

### **Purchasing**

By the end of this course, students will:

- identify and describe various incentives in making purchasing decisions (e.g., 20% off;  $\frac{1}{3}$  off; buy 3 get 1 free; loyalty rewards; coupons; 0% financing);
- estimate the sale price before taxes when making a purchase (e.g., estimate 25% off of \$38.99 as 25% or  $\frac{1}{4}$  off of \$40, giving a discount of about \$10 and a sale price of approximately \$30; alternatively, estimate the same sale price as about  $\frac{3}{4}$  of \$40);
- describe and compare a variety of strategies for estimating sales tax (e.g., estimate the sales tax on most purchases in Ontario by estimating 10% of the purchase price and adding about a third of this estimate, rather than estimating the PST and GST separately), and use a chosen strategy to estimate the after-tax cost of common items (**Sample problem:** You purchase three items for \$8.99 each and one item for \$4.99. Estimate the after-tax total.);
- calculate discounts, sale prices, and after-tax costs, using technology;
- identify forms of taxation built into the cost of an item or service (e.g., gasoline tax, tire tax);
- estimate the change from an amount offered to pay a charge (**Sample problem:** Estimate the change from the \$20 offered to pay a charge of \$13.87.);
- make the correct change from an amount offered to pay a charge, using currency manipulatives (**Sample problem:** Use currency manipulatives to explain why someone might offer \$15.02, rather than \$15.00, to pay a charge of \$13.87.);
- compare the unit prices of related items to help determine the best buy (**Sample problem:** Investigate whether or not purchasing larger quantities always results in a lower unit price.);
- describe and compare, for different types of transactions, the extra costs that may be associated with making purchases (e.g., interest costs, exchange rates, shipping and handling costs, customs duty, insurance) (**Sample problem:** What are the various costs included in the final total for purchasing a digital audio player online from an American source? Using an online calculator, calculate the final cost, and describe how it compares with the cost of the purchase from a major retailer in Ontario.);
- make and justify a decision regarding the purchase of an item, using various criteria (e.g., extra costs, such as shipping costs and transaction fees; quality and quantity of the item; shelf life of the item; method of purchase, such as online versus local) under various circumstances (e.g., not having access to a vehicle; living in a remote community; having limited storage space) (**Sample problem:** I have to take 100 mL of a liquid vitamin supplement every morning. I can buy a 100 mL size for \$6.50 or a 500 mL size for \$25.00. If the supplement keeps in the refrigerator for only 72 h, investigate which size is the better buy. Explain your reasoning.).



## Saving, Investing, and Borrowing

### Overall Expectations

By the end of this course, students will:

- describe and compare services available from financial institutions;
- demonstrate an understanding of simple and compound interest, and solve problems involving related applications;
- interpret information about different ways of borrowing and their associated costs, and make and justify informed borrowing decisions.

### Specific Expectations

#### *Comparing Financial Services*

By the end of this course, students will:

- gather, interpret, and compare information about the various savings alternatives commonly available from financial institutions (e.g., savings and chequing accounts, term investments), the related costs (e.g., cost of cheques, monthly statement fees, early withdrawal penalties), and possible ways of reducing the costs (e.g., maintaining a minimum balance in a savings account; paying a monthly flat fee for a package of services);
- gather, interpret, and compare information about the costs (e.g., user fees, annual fees, service charges, interest charges on overdue balances) and incentives (e.g., loyalty rewards; philanthropic incentives, such as support for Olympic athletes or a Red Cross disaster relief fund) associated with various credit cards and debit cards;
- read and interpret transaction codes and entries from various financial statements (e.g., bank statement, credit card statement, passbook, automated banking machine printout, online banking statement, account activity report), and explain ways of using the information to manage personal finances (**Sample problem:** Examine a credit card statement and a bank statement for one individual, and comment on the individual's financial situation.).

#### *Saving and Investing*

By the end of this course, students will:

- determine, through investigation using technology (e.g., calculator, spreadsheet), the effect on simple interest of changes in the principal, interest rate, or time, and solve problems involving applications of simple interest;
- determine, through investigation using technology, the compound interest for a given investment, using repeated calculations of simple interest for no more than 6 compounding periods (**Sample problem:** Someone deposits \$5000 at 4% interest per annum, compounded semi-annually. How much interest accumulates in 3 years?);
- describe the relationship between simple interest and compound interest in various ways (i.e., orally; in writing; using tables and graphs);
- determine, through investigation using technology (e.g., a TVM Solver in a graphing calculator or on a website), the effect on the future value of a compound interest investment of changing the total length of time, the interest rate, or the compounding period (**Sample problem:** Compare the results at age 40 of making a deposit of \$1000 at age 20 or a deposit of \$2000 at age 30, if both investments pay 6% interest per annum, compounded monthly.);

- solve problems, using technology, that involve applications of compound interest to saving and investing.

### ***Borrowing***

By the end of this course, students will:

- gather, interpret, and compare information about the effects of carrying an outstanding balance on a credit card at current interest rates (***Sample problem:*** Describe ways of minimizing the cost of carrying an outstanding balance on a credit card.);
- gather, interpret, and compare information describing the features (e.g., interest rates; flexibility) and conditions (e.g., eligibility; required collateral) of various personal loans (e.g., student loan; car loan; “no interest” deferred-payment loan; loan to consolidate debt; loan drawn on a line of credit; payday or bridging loan);
- calculate, using technology (e.g., calculator, spreadsheet), the total interest paid over the life of a personal loan, given the principal, the length of the loan, and the periodic payments, and use the calculations to justify the choice of a personal loan;
- determine, using a variety of tools (e.g., spreadsheet template, online amortization tables), the effect of the length of time taken to repay a loan on the principal and interest components of a personal loan repayment;
- compare, using a variety of tools (e.g., spreadsheet template, online amortization tables), the effects of various payment periods (e.g., monthly, biweekly) on the length of time taken to repay a loan and on the total interest paid;
- gather and interpret information about credit ratings, and describe the factors used to determine credit ratings and the consequences of a good or bad rating;
- make and justify a decision to borrow, using various criteria (e.g., income; cost of borrowing; availability of an item; need for an item) under various circumstances (e.g., having a large existing debt; wanting to pursue an education or training opportunity; needing transportation to a new job; wanting to set up a business).

## Transportation and Travel

### Overall Expectations

By the end of this course, students will:

- interpret information about owning and operating a vehicle, and solve problems involving the associated costs;
- plan and justify a route for a trip by automobile, and solve problems involving the associated costs;
- interpret information about different modes of transportation, and solve related problems.

### Specific Expectations

#### *Owning and Operating a Vehicle*

By the end of this course, students will:

- gather and interpret information about the procedures (e.g., in the graduated licensing system) and costs (e.g., driver training; licensing fees) involved in obtaining an Ontario driver's licence, and the privileges and restrictions associated with having a driver's licence;
- gather and describe information about the procedures involved in buying or leasing a new vehicle or buying a used vehicle;
- gather and interpret information about the procedures and costs involved in insuring a vehicle (e.g., car, motorcycle, snowmobile) and the factors affecting insurance rates (e.g., gender, age, driving record, model of vehicle, use of vehicle), and compare the insurance costs for different categories of drivers and for different vehicles (**Sample problem:** Use automobile insurance websites to investigate the degree to which the type of car and the age and gender of the driver affect insurance rates.);
- gather and interpret information about the costs (e.g., monthly payments, insurance, depreciation, maintenance, miscellaneous expenses) of purchasing or leasing a new vehicle or purchasing a used vehicle, and describe the conditions that favour each alternative (**Sample problem:** Compare the costs of buying a new car, leasing the same car, and buying an older model of the same car.);
- describe ways of failing to operate a vehicle responsibly (e.g., lack of maintenance, careless driving) and possible financial and non-financial consequences (e.g., legal costs, fines, higher insurance rates, demerit points, loss of driving privileges);
- identify and describe costs (e.g., gas consumption, depreciation, insurance, maintenance) and benefits (e.g., convenience, increased profit) of owning and operating a vehicle for business (**Sample problem:** Your employer pays 35 cents/km for you to use your car for work. Discuss how you would determine whether or not this is fair compensation.);
- solve problems, using technology (e.g., calculator, spreadsheet), that involve the fixed costs (e.g., licence fee, insurance) and variable costs (e.g., maintenance, fuel) of owning and operating a vehicle (**Sample problem:** The rate at which a car consumes gasoline depends on the speed of the car. Use a given graph of gasoline consumption, in litres per 100 km, versus speed, in kilometres per hour, to determine how much gasoline is used to drive 500 km at speeds of 80 km/h, 100 km/h, and 120 km/h. Use the current price of gasoline to calculate the cost of driving 500 km at each of these speeds.).

**Travelling by Automobile**

By the end of this course, students will:

- determine distances represented on maps (e.g., provincial road map, local street map, Web-based maps), using given scales (**Sample problem:** Compare the driving distances between two points on the same map by two different routes.);
- plan and justify, orally or in writing, a route for a trip by automobile on the basis of a variety of factors (e.g., distances involved; the purpose of the trip; the time of year; the time of day; probable road conditions; personal priorities);
- report, orally or in writing, on the estimated costs (e.g., gasoline, accommodation, food, entertainment, tolls, car rental) involved in a trip by automobile, using information from available sources (e.g., automobile association travel books, travel guides, the Internet);
- solve problems involving the cost of travelling by automobile for personal or business purposes (**Sample problem:** Determine and justify a cost-effective delivery route for ten deliveries to be made in a given area over two days.).

**Comparing Modes of Transportation**

By the end of this course, students will:

- gather, interpret, and describe information about the impact (e.g., monetary, health, environmental) of daily travel (e.g., to work and/or school), using available means (e.g., car, taxi, motorcycle, public transportation, bicycle, walking) (**Sample problem:** Discuss the impact if 100 students decided to walk the 3 km distance to school instead of taking a school bus.);

- gather, interpret, and compare information about the costs (e.g., insurance; extra charges based on distance travelled) and conditions (e.g., one-way or return; drop-off time and location; age of the driver; required type of driver's licence) involved in renting a car, truck, or trailer, and use the information to justify a choice of rental vehicle (**Sample problem:** You want to rent a trailer or a truck to help you move to a new apartment. Investigate the costs and describe the conditions that favour each option.);
- gather, interpret, and describe information regarding routes, schedules, and fares for travel by airplane, train, or bus;
- solve problems involving the comparison of information concerning transportation by airplane, train, bus, and automobile in terms of various factors (e.g., cost, time, convenience) (**Sample problem:** Investigate the cost of shipping a computer from Thunder Bay to Windsor by airplane, train, or bus. Describe the conditions that favour each alternative.).

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